Topical White Paper for the Decadal Survey on Biological and Physical Sciences (BPS) Research in Space 2023-2032 (BPS2023), The National Academies of Sciences, Engineering, and Medicine

Short-range Measurements of the Gravitational Inverse Square Law in the Micro-gravity Environment

Thematic Areas:
☒ Gravitational and other space environment effects on physical and biological processes involved in the functioning of space exploration technologies.
☒ The effects of the spaceflight environment, including gravitational effects, on physical systems and processes.
☐ The effects of the spaceflight environment on biological and biophysical systems and processes.

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Abstract: The accelerating expansion of the universe and galaxy rotation curves, which are described by dark energy and dark matter, could be related to a modification of gravity on large scales. As ground-based tests of Newtonian inverse square law of gravity are affected by the Earth gravity and terrestrial noise, it is essential to perform these experiments in spaceborne laboratories using drive and test masses isolated from any terrestrial and seismic noise. In-situ micro-gravity measurements of the inverse square law will help us comprehend better gravitational phenomena at different scales.

Introduction – Contemporary discoveries of the accelerating expansion of large-scale structures [1–3] and the radial velocity profiles of spiral galaxies [4–7] are in contraction to the assumptions of Newtonian gravity. Although these observations are widely ascribed to dark energy [8–11] and dark matter [12–15] in the ΛCDM standard model of cosmology, they could also be explained by a modification of the gravitational laws in the low-energy (infrared) limit on large scales (see e.g. [16–18]) such as theories of massive gravity (see reviews by [19, 20]), which could be analogous to the electromagnetic behaviors in the superconducting phase formulated by the Maxwell–Proca equations of massive photons [21, 22]. In particular, it is theoretically possible to map a 4-dimensional Yang-Mills theory onto a gravitational theory in $4 + 1$ spacetime based on the holographic principle [23, 24] and gauge/gravity duality [25–27]. Thus, we may also have a geometric counterpart of the Brout-Englert-Higgs (BEH) mechanism [28–30] of Yang-Mills theories that could lead to a massive gravity in the infrared completion [31–33].

The theory of general relativity, which is considered as the most acceptable geometric description of gravity, has been confirmed in several observations, such as recent detections of gravitational waves from merged compact binaries [34–37], and very-long-baseline interferometry reconstructed imaging of a supermassive black hole [38]. Moreover, general-relativistic frame-dragging effects (see e.g. [39–41]) were also measured by two artificial satellites orbiting the Earth [42], and observations of a binary system [43]. Several ground-based experiments have also validated the equivalence principle [44–48] and Newtonian inverse square law at various scales [49–53]. While general relativity is found to accurately describe gravitational phenomena in the high-energy limit around compact objects, Newtonian gravity is still valid in our Earth-based everyday experiments. Nowadays, recent technological advancements allow us to precisely measure the inverse-square law of gravity at μm scales, which were not previously possible in the experiment performed by Henry Cavendish [54] in the 18th century. Several Earth-bound experiments of Newton's inverse square law (for example, see Fig. 1) have been carried out with small test and drive masses [51–53, 55–57]. However, the accuracy of our ground-based measurements is limited by the Earth gravity and seismic noise, which make it extremely difficult to reach a conclusive finding about gravitational interactions between small source masses at short scales in the low-energy situation. We do not know whether general relativity is still valid on cosmological scales of galaxies and on μm scales in μ-g empty spaces in the low-energy limit.

Departure from Newtonian gravity – When Newton's inverse square law of gravity was introduced in 1687 [58], no experiment has been performed to verify it until the Cavendish experiment in 1797 [54]. Newton's inverse square law is described by

$$F_{\mathrm{N}}(r) = G_{\mathrm{N}}\frac{m_1 m_2}{r^2}, \tag{1}$$

where F_{N} is the Newtonian gravitational force acting between two masses m_1 and m_2 separated by distance r, and G_{N} is the Newtonian gravitational constant.

A possible departure from Newtonian gravity was proposed by Fujii in 1971 [59] by assuming a dilaton-meditated gravitational force below a short distance r_{N}:

$$F(r) = \frac{3}{4}G_{\mathrm{N}}\frac{m_1 m_2}{r^2}\left[1 + \frac{1}{3}e^{-r/\lambda}\left(1 + r/\lambda\right)\right], \tag{2}$$

where λ is an interaction range for the dilaton-type field. Newtonian gravity recovers at large

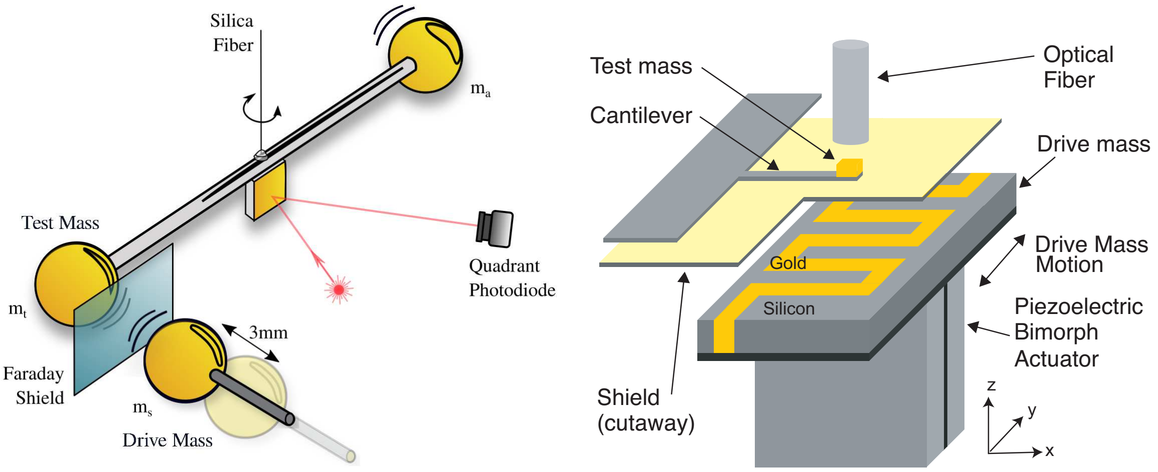

Figure 1: Schematic of the gravitational inverse square law experiments. *Left Panel*: The torsion-balance Cavendish-type experiment conducted by Ref. [57]. *Right Panel*: Yukawa-type force cantilever-based measurement performed by Ref. [55] using a gold test mass on a cantilever above a drive mass consisting of a gold meander pattern inside a silicon substrate. Thick conductive Faraday shields suppress electric couplings.

distances $\lambda \gg r_{\rm N}$. Following Fujii's proposal, some laboratory experiments have been performed to find a dependence of the gravitational constant on r between two masses [60].

A generalized from of Fujii's expression is given by a Yukawa-type gravitational force (see reviews by [50, 61, 62]) as follows:

$$F(r) = \frac{1}{1+\alpha} G_{\rm N} \frac{m_1 m_2}{r^2} \left[1 + \alpha e^{-r/\lambda} \left(1 + r/\lambda\right)\right], \tag{3}$$

where λ is the Yukawa range written as $\lambda = \hbar/(m_{\rm G} c)$, α is the Yukawa dimensionless parameter associated with the strength of the modified gravity with respect to Newtonian gravity (note $\alpha = 1/3$ yields Fujii's expression), and $m_{\rm G}$ is the graviton mass. If the graviton acquires mass through a Higgs-like mechanism in the infrared (low-energy) limit (see e.g. [31–33, 63]), the gravitational force takes the Yukawa-type form of Eq. (3), whereas Newtonian gravity of Eq. (1) is still valid in the ultraviolet (high-energy) limit ($m_{\rm G} \to 0$).

Additionally, a power-law gravitational force modified by arbitrary powers N has also been suggested [64]:

$$F(r) = \frac{1}{1+\alpha_{\rm N}} G_{\rm N} \frac{m_1 m_2}{r^2} \left[1 + \alpha_{\rm N} N \left(\frac{r_0}{r}\right)^{N-1}\right], \tag{4}$$

where $\alpha_{\rm N}$ is a dimensionless constant, and r_0 is the length scale of a non-Newtonian force. For example, the simultaneous interactions through two massless scalar and two massless pseudoscalar particles are produced using $N = 2$ and 3, respectively. In the limit $N \to 1$, one obtains the Newtonian gravitational force of Eq. (1).

Considering the Yukawa-type gravitational force described by Eq. (3), the departure from Newtonian gravity can be characterized by only the two parameters: the Yukawa range λ and dimensionless parameter α. Various constraints on the Yukawa parameters on large scale ($\lambda \geq 10^{-3}$ m) from laboratory, geophysical measurements, Earth-LAGEOS-Lunar experiments, lunar-laser-ranging (LLR) measurements, and planetary observations were summarized by Ref. [61]

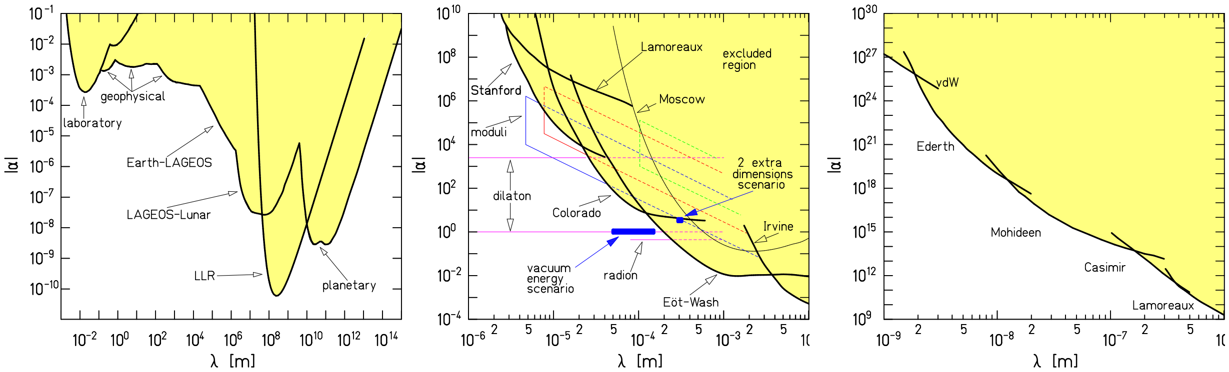

Figure 2: Constraints on the Yukawa dimensionless parameter ($|\alpha|$) versus the Yukawa range (λ) at long ranges ($\lambda > 10^{-3}$ m; left panel), medium ($10^{-6} < \lambda < 10^{-2}$ m; left panel), and short ($\lambda < 10^{-6}$ m; right panel), from [50].

(Chapter 2), which is shown in Figure 2 (left panel; included in the review by [50]). We should note that the lunar orbit precession is due to perturbations from Earth and other solar planets, and a small fraction owing to general relativity. The results from medium-range experiments ($10^{-6} < \lambda < 10^{-2}$ m) are also given in Figure 2 (middle panel). 95%-confidence level constraints on λ and α from 1 nm to 10 μm are also presented in the right panel of Figure 2, where the departure from Newtonian gravity strongly follows a power-law modification expressed by Eq. (4).

Figure 1 shows the two recent different setups of gravitational inverse square law experiments at short ranges performed using: (1) a Cavendish-type torsion experiment [57], and (2) a cantilever-based experiment instead of torsion balances [55]. In the first experiment [57], they measured gravitational interaction between two gold balls of 1 mm radius and sub-100-milligram mass. This experiment is a first step towards quantum measurements of gravity between sub-mm-sized masses of 10^{-13} gram using current quantum sensing technologies (see e.g. [65, 66]). In the second experiment [55], they build an apparatus for cantilever-based experiments to measure attonewton-scale gravitational interaction between gold drive and test masses separated by 25 μm. This experiment, which was implemented by a gold test mass on a cantilever over a drive mass made by a gold meander pattern inside a silicon substrate, puts new constraints on the parameter space of λ–α of Yukawa-type deviations from Newtonian gravity in the range of 6–20 μm (see Fig. 16 in [55], compared to Figure 2 middle). This kind of cantilever-based experiments can provide new constraints for quantum gravity and graviton mass. However, the most sensitive Earth-based Cavendish-type experiments are still limited by seismic and terrestrial noise, so space-based experiments should definitely provide us with a much better sensitivity level.

Proposed micro-gravity drag-free experiments – Future short-range measurements of the inverse square law of gravity in the μg environment will improve our constraints on the λ–α parameter space, which allow us to constrain Yukawa-type and power-law modifications of gravity specified by Eqs. (3) and (4), and finally gravitational interaction on various scales. Currently, on-ground experiments are contaminated by the Earth gravity and seismic noise. In particular, Cavendish-type torsion experiments are affected by mechanical and thermal noise from the hanging wire or fiber, while cantilever-based experiments could also include some thermal noise from cantilevers. In the micro-gravity environment without the Earth gravity, there is no need for wire, so one could precisely measure gravitational interactions between small masses at smaller ranges. Space-based inverse square law experiments offer new possibilities to detect a deviation from Newtonian gravity on small scales in a drag-free satellite in the micro-gravity environment.

Currently, a similar proposed spaceborne experiment known as STEP (Satellite Test of the Equivalence Principle) [67] is under development for very sensitive gravitational research that will examine the equivalence principle with a sensitivity level at five orders of magnitude better than the ground-based one. This experiment in a drag-free satellite is entirely free from terrestrial sources of noise such as seismic effects. As the micro-gravity environment has less noise owing to gas pressures, cryogenic technologies can also be employed in experiments, such as a Superconducting Quantum Interference Device (SQUID) position detector. Previously, a spaceborne satellite for searching deviations from Newtonian gravity named SEE (Satellite Energy Exchange) was proposed in 1992 [68–70], which was aimed at testing both the equivalence principle and inverse square law of gravity. In addition to STEP and SEE, other spaceborne experiments have been proposed prior to 1992 for exploring deviations from Newtonian gravity [71–76], which have also been argued for the absence of terrestrial noise in space-based measurements. However, none of them have been deeply studied and developed later, except for the STEP experiment.

The methods and concepts developed and studied for the STEP experiment [67], together with new technologies recently developed for ground-based experiments (e.g. [51–53, 55, 57]) can help us design a practical spaceborne apparatus for measurements of the gravitational inverse square law in the μg environment inside a drag-zero satellite. All the setups and technologies utilized in ground-based experiments have been reviewed by Ref. [62] (in Section 5).

Conclusion – In summary, we have proposed to consider practical spaceborne experiments for constraining deviations from Newtonian gravity on short scales. This experiment can help us search for non-Newtonian gravity that provides new insights into gravitational phenomena on various scales. The proposed micro-gravity short-range measurements of the inverse square law will prepare us for verifying different theories of modified Newtonian gravity such as massive gravity on large scales, where our observations of galactic-scale structures are currently supposed to be made by dark energy and dark matter in the ΛCDM cosmological model.

If our future spaceborne experiments reveal a deviations from Newton's inverse square law at a short range, we should investigate whether it is associated with extra dimensions or a massive graviton. It will also be required to examine whether the equivalence principle is still valid at such a short range. Moreover, it will be essential to test the inverse square law on large scales. As proposed by Ref. [77], the Laser Interferometer Space Antenna mission (LISA; planned to be launched in 2034) should be able to test a Yukawa-type gravity of massive gravity at a scale of 6×10^{19} m using gravitational waves (GW) produced by inspiraling super-massive black hole binaries. Moreover, GW in massive gravity could also carry three extra polarization modes (two vectors and one scalar; see e.g. [78]), in addition to two tensorial polarization modes tensors predicted by general relativity (see also discussion by [63]). These three extra GW polarization modes can be detected using 3-arm space-borne GW detectors such as the LISA. Thus, future spaceborne measurements of the inverse square law and GW observations by the LISA will enhance our understanding of gravity on short and large scales.

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