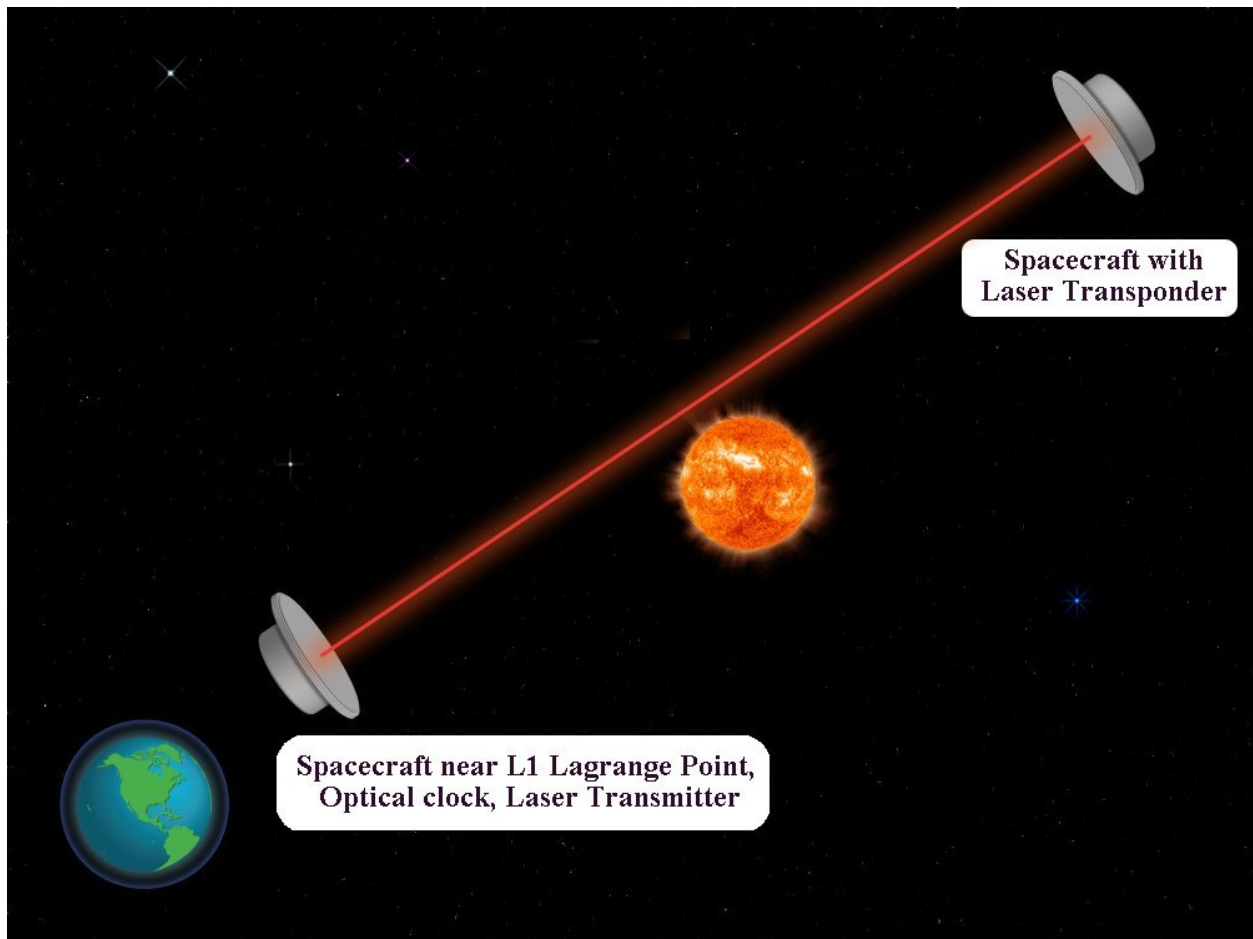


## **Topical: Testing Gravitational Time Delay Predictions of General Relativity Response to Call for White Papers for Decadal Survey**

Neil Ashby,\* Dept. of Physics, UCB 390,  
University of Colorado, Boulder, CO 80309-0390.  
Peter Bender, JILA, University of Colorado, Boulder, CO 80309  
Bijunath Patla, National Institute of Standards and Technology,  
Boulder, CO, 80305



### **Testing Gravitational Time Delay Predictions of General Relativity**

\* e-mail: [ashby@boulder.nist.gov](mailto:ashby@boulder.nist.gov); Phone: (303) 499-2145

**Introduction:** Testing general relativity (GR) is one of the important scientific opportunities for new missions and new observation programs in the next decade. Time delay measurement is a definitive test to determine whether gravity is a purely geometric entity. Deviations from the predictions of GR can be expressed in terms of the Parameterized Post-Newtonian (PPN) formulation of gravitational theory, in which the main contribution to time delay of electromagnetic waves passing near a massive object is proportional to  $1 + \gamma$ , where  $\gamma$  is a measure of the curvature of space-time, and is equal to unity in GR. The current best measurement of  $\gamma^* = \gamma - 1$  with  $2.3 \times 10^{-5}$  accuracy was made during the Cassini mission.[1] Further improvements in the accuracy for  $\gamma^*$  to roughly  $1 \times 10^{-6}$  are expected from two missions of the European Space Agency (ESA): The GAIA astrometric mission, which will measure the gravitational deflection of light rays by the Sun, and the BepiColombo mission to Mercury, which will make improved measurements of the solar time delay. Here we discuss a mission that can measure  $\gamma^*$  to roughly  $1 \times 10^{-8}$ .

In 2005, a proposal was first made for measurements between a spacecraft near the L1 point of the Earth-Sun system to a transponder spacecraft that would pass behind the Sun.[2] This proposal has been updated to include the use of extremely stable new clocks based on optical transitions in cooled atoms or ions.[3] It provides an example of an approach that would be needed to reach an accuracy of about  $1 \times 10^{-8}$  for determining  $\gamma$ .

**Science Goals:** In view of the well-known lack of a theory that connects GR with quantum theory, an important goal is to improve high-accuracy tests of the predictions of gravitational theory. An approach that would give about two orders of magnitude improvement in tests of GR is through improved gravitational time delay measurements. However, other improvements in earlier tests and completely new types of tests are expected to occur during the next decade. Recent increases in the accuracy for lunar laser range measurements have substantially improved accuracy in tests of Einstein's Strong Equivalence Principle to a present level of  $3 \times 10^{-4}$ . While observations of gravitational waves confirm the dynamical predictions of GR under very high field conditions, there is still a possibility that a scalar-tensor theory may be confirmed and hence that gravitation is not purely a metric phenomenon.

There is a strong reason to believe that highly accurate measurements of the gravitational time delay may be the most promising method for detecting deviations from general relativity. Many alternatives to general relativity involve additional scalar fields. Studies of the evolution of scalar fields in the matter-dominated era of the universe indicate that the universe's expansion tends to drive the scalar fields toward a state in which the scalar-tensor theory is only slightly different from GR. Some scalar-tensor extensions of GR predict deviations from general relativity in the range of  $\gamma^*$  from  $10^{-5} - 10^{-8}$  [4, 5]. Improved information about  $\gamma$  would provide important insight into the evolution of the universe and directly limit the range of applicability of alternative gravitational theories.

**Description of the reference mission orbits and predicted time delay:** For the reference mission, one spacecraft (S1) containing a highly stable optical clock would be placed in an orbit around the L1 point, about 1.5 million km from the Earth in the direction of the Sun. The second spacecraft (S2) would be placed in a 2-year period orbit in the ecliptic plane, with an eccentricity

of 0.37. S2 would pass through superior solar conjunction, as seen from L1, about one year after launch and 2 and 4 years thereafter. The spacecrafts would have carefully designed drag-free systems to nearly eliminate the effects of spurious non-gravitational forces on them. A measurement of  $\gamma - 1$  to a level of  $10^{-8}$  could be carried out by observing the time delay of laser signals exchanged between two spacecraft when the line of sight passes near the sun's limb. Atmospheric effects would be absent. Also, with a 2-yr orbit period for the distant satellite S2, at aphelion the spacecraft temperature will not change much during an 8-day observing period. By adjusting the phase of the S2 orbit with respect to the earth, the aphelion of the S2 orbit can be made to occur during the measurements; the range rate then becomes very close to zero, and the orbit determination problem is much reduced.

The crucial measurements of time delay occur within a few days of superior conjunction and are primarily characterized by a logarithmic dependence on the distance of closest approach of the light to the mass source. The predicted gravitational time delay due to a non-rotating mass source (here  $\mu = GM / c^2$ ), expressed in terms of the radii  $r_A, r_B$  of the endpoints of the photon, and the elongation angle  $\Phi$  between the radius vectors from the source to the endpoints, is [6]

$$c\Delta t_{\text{delay}} = \mu(1 + \gamma) \log \left[ \frac{r_A + r_B + r_{AB}}{r_A + r_B - r_{AB}} \right] - \frac{\mu^2(1 + \gamma)^2 r_{AB}}{r_A r_B (1 + \cos \Phi)} + \frac{\mu^2 r_{AB} (8 - 4\beta + 8\gamma + 3\epsilon)}{4r_A r_B \sin \Phi}. \quad (1)$$

where  $r_{AB}$  is the geometric distance between the endpoints in isotropic coordinates and  $\beta, \epsilon$  are PPN parameters measuring the nonlinearity of the time-time and space-space components of the metric tensor. In GR,  $(8 - 4\beta + 8\gamma + 3\epsilon) / 4 = 15 / 4$ . The time delay in the above equation is expressed in terms of observable quantities, or quantities that can be obtained from orbit determination programs and does not involve the unknown impact parameter or distance of closest approach. The non-linear terms are a few nanoseconds so are significant, but do not have to be estimated with great accuracy. The second-order terms are enhanced near superior conjunction when the elongation angle  $\Phi$  comes close to  $\pi$ . However Eq. (1) is only applicable if  $\Phi$  differs from  $\pi$  by enough so that the line of sight does not pass through the sun. The time delay near superior conjunction is plotted in Figure 1.

The time delay due to the solar quadrupole moment is controlled by the parameter

$$\frac{\mu J_2 R^2}{b^2 c} < 10^{-12} \text{ s} \quad (2)$$

where  $J_2 \approx 2 \times 10^{-7}$ ,  $R$  is the sun's radius, and  $b \approx R$  is the distance of closest approach. There is a complicated dependence of this delay on the orientation of the sun's rotation axis with respect to the photon path, but the net effect is only a few ps and can be estimated with sufficient accuracy that it will not contribute significantly to the error budget.

The measurements will be made by transmitting a cw laser beam from S1 to S2, phase locking the laser signal from the transponder on the distant spacecraft to the received beam, and then recording the relative phase of the received beam back at S1 with respect to its own laser as

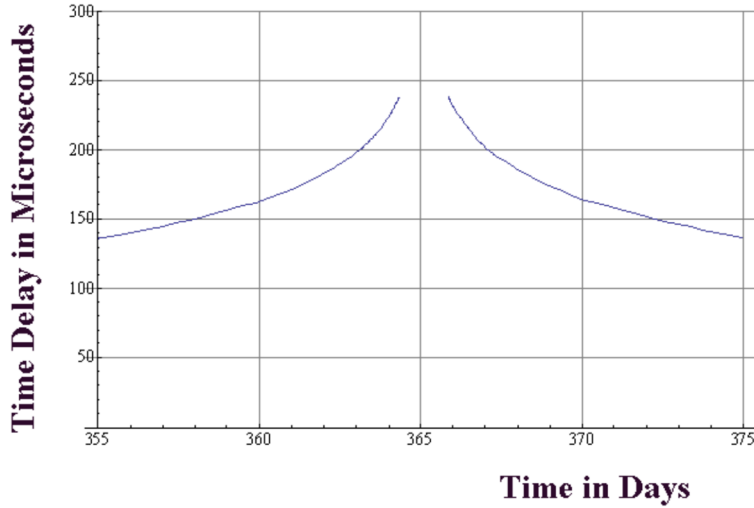


Figure 1. Round-trip time delay of light as a function of time elapsed after launch. The logarithmic dependence of the delay gives a unique time signature. The region within 0.75 days of conjunction is excluded due to occultation by the sun.

a function of time. With 20 cm diameter telescopes, and given the round-trip travel time of about 3000 s, the received signal would be roughly 1000 counts/s for 1 W of transmitted power. This is a weak signal, but it is strong enough so that the chances of a cycle slip should be very small, provided that the laser in S2 is well stabilized. If we consider the round-trip delay times  $\Delta t_{\text{delay}}$  to be the observable, then the change in delay from 0.75 days to 4 days on either side of conjunction is about 64 microseconds.

**Signal-to-Noise Analysis:** We estimate the final uncertainty that can be attained in this experiment based on the Optimal Wiener filter, which takes advantage of the known time signature of the signal and includes the expected noise sources.[7] For simplicity, the distance between the spacecraft is assumed to be constant except for the gravitational time delay. The time signature of  $\gamma^* = (1 + \gamma) / 2$  is taken to be represented by the logarithmic function

$$g(t) = -B(\log|Rt| - M), \quad (3)$$

where  $M = \langle \log|Rt| \rangle = (t_2 \ln(Rt_2) - t_1 \ln(Rt_1)) / (t_2 - t_1) - 1$ . is the mean value of  $\log|Rt|$  over the time periods  $-t_2$  to  $-t_1$  and  $+t_1$  to  $+t_2$  (a short time interval during occultation is excluded), and for the proposed experiment  $B = 0.97 \times 8\mu / c = 3.82 \times 10^{-5} s$ . The rate at which the line of sight to the distant spacecraft passes across the sun is  $R = 1.9$  solar radii per day.

Let  $g(f)$  be the one-sided Fourier transform of  $g(t)$  over the time of the measurements. Then the square of the signal-to-noise ratio is given by

$$\left(\frac{S}{N}\right)^2 = \int_0^{\infty} \frac{2|g(f)|^2}{n(f)^2} df, \quad (4)$$

For example, if the time interval of measurements extends from 0.75 days to 4 days, Figure 2 plots the quantity  $g(f)^2 / B^2$ . When combined with estimates of the spectral density of the noise, the function gives us the signal to noise ratio according to Eq. (4) above. If the noise has a constant spectral density, only about 5% of the integral in Eq. (4) comes from frequencies below 1 microhertz, where the power spectral density of acceleration noise is expected to increase.

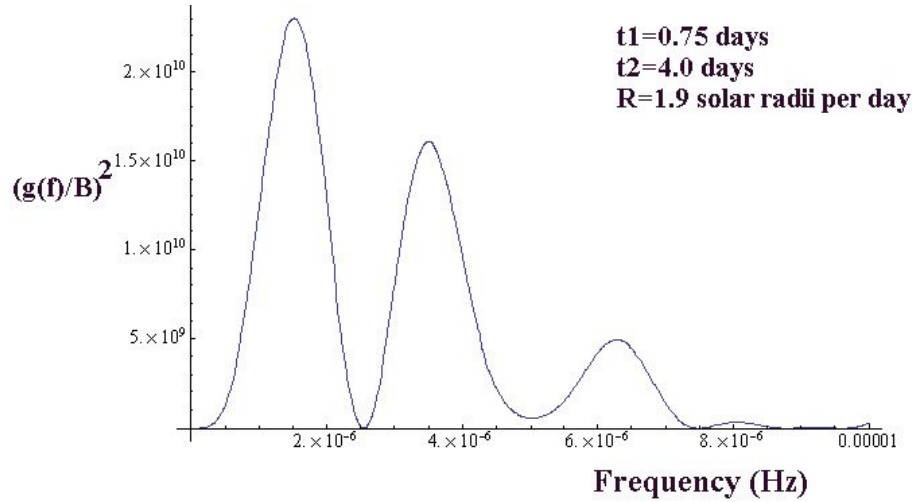


Figure 2. Plot of the function  $g(f)^2 / B^2$  for frequencies up to 0.01 mHz, for a typical set of measurement parameters.

**S1 spacecraft clock:** The major requirement is to fly an optical clock on S1 that has very high stability over at least 8 days around superior conjunction. The goal is to achieve a fractional frequency noise power spectral density amplitude of  $3 \times 10^{-15} / \sqrt{\text{Hz}}$  from 1 Hz down to at least  $10^{-6}$  Hz. (This is approximately equivalent to an Allan deviation of  $5 \times 10^{-15} / \sqrt{\tau}$  for periods  $\tau$  from 1 s to up to  $10^6$  s.) A leading candidate for the optical clock is an Ytterbium positive ion clock based on a single cooled  $\text{Yb}^+$  ion in a trap. The projected spectral density amplitude is about  $3 \times 10^{-15} / \sqrt{\text{Hz}}$  over the relevant frequency range [8]. An advantage of  $\text{Yb}^+$  is that only low power lasers are required. However, substantial development is needed to show that such lasers can be space qualified. Alternate choices include trapped ion clocks based on  $\text{Sr}^+$  or  $\text{Al}^+$ .

The measurements of time delay over at least the 8 days around superior conjunction would be made as follows. The laser beam from the clock on S1 would be expanded to about 20 cm diameter with a Cassegrain telescope and then transmitted to S2. The beam arriving at S2 would be received by a similar 20 cm telescope and heterodyne detected against a prestabilized on-board laser with nearly the same wavelength. The S2 laser would be phase-locked to the received beam with a small frequency offset, then retransmitted back to S1. There the relative phase of the received signal would be recorded as a function of time. The relative phase would be strongly affected by the relative motion of the two spacecraft in their orbits, and thus all the relevant orbit parameters, as well as  $\gamma$ , would have to be solved for.

In the time delay measurements, care will be needed to reduce the effects of scattered light in the telescopes. But with heterodyne detection, this does not appear to be a serious problem. The plan for such a mission would be to not make measurements closer than about 0.4 solar radii to the limb, to avoid problems with reacquisition of the signals after solar conjunction.

**Drag-free system:** The required performance builds on that planned for the LISA gravitational wave mission. The main challenge is minimizing thermal changes, and particularly thermal gradient changes, near the freely floating test mass that is the heart of the drag-free system. On LISA this is done almost completely by passive thermal isolation. For a time delay mission, a slow active temperature control system would be used at frequencies below  $10^{-4}$  Hz. Changes in solar heat input over the 8 days around superior solar conjunction will be quite small, even for the elliptical orbit of S2, because it would be near aphelion at that time. The required drag-free performance is roughly  $1 \times 10^{-13} \text{ m/s}^2 / \sqrt{\text{Hz}}$  down to  $10^{-6}$  Hz.

The overall performance of a drag-free system was demonstrated in the LISA Pathfinder Mission, which was launched in 2015.[9] The requirements are just to demonstrate an acceleration spectral density amplitude of  $3 \times 10^{-14} \text{ m/s}^2 / \sqrt{\text{Hz}}$  performance down to  $10^{-3}$  Hz, because of the less ideal thermal stability expected for LISA Pathfinder, compared with LISA. The mission[9] demonstrated that two free-falling reference test masses can have a relative acceleration noise power spectral density of  $5.2 \times 10^{-15} \text{ m/s}^2$  for frequencies between 0.7 and 20 mHz. Below 0.5 mHz the Pathfinder mission exhibited a low-frequency tail that remained below  $12 \times 10^{-15} \text{ m/s}^2$  down to 0.1 mHz. Thus the design of the drag-free systems for a gravitational time delay mission does not appear to be a substantial limitation. Both the distant spacecraft S2 and the S1 spacecraft would have very low levels of non-gravitational orbit disturbances.

**Other scientific benefits from proposed mission:** Effects such as those arising from non-linear terms in the metric, parameterized by  $\beta$ , as well as other time delay effects originating in the sun's rotation, can also be measured. The spacecraft at L1 will be subject to numerous relativistic effects that alter the frequency of the optical clock relative to a reference on earth. These effects include the potentials of the sun, moon, and other solar system bodies, and second-order time dilation effects due to velocities of earth and spacecraft. Such effects cause the frequency to vary from that of a clock at an infinite distance from earth, which is characterized by the parameter  $L_G = -6.969290134 \times 10^{-10}$ . The frequency difference varies around  $L_G$  by a small amount, due mainly to earth's orbital eccentricity. This should be measurable to better than a part per million. Examples of other effects include that of the moon ( $\approx 10^{-13}$ ) or Jupiter ( $\approx 10^{-15}$ ) which are also measurable with an optical clock of stability.

**Conclusion:** Adoption of a major science goal of improving high-accuracy tests of the predictions of gravitational theory is recommended. Then a promising type of test is a roughly two orders of magnitude improvement in gravitational time delay measurements.

## References

- [1] Bertotti, L. Iess, and P. Tortora, *Nature* **425**, 374, (2003);  
<https://doi.org/10.1038/nature01997>
- [2] P. Bender and N. Ashby in “Lasers, Clocks, and Drag-Free Control,” Bremen, Germany, June 2005 *Astrophysics and Space Science Library* 349, eds. H. Dittus, C. Laemmerzahl, S. Turyshev, Springer (2008), pp 219-230.
- [3] P. Bender, N. Ashby, R. Dolesi, *et al.*, “Quantum to Cosmos III Workshop,” Warrenton, VA, July 6-10, 2008.
- [4] T. Damour and G. Esposito-Farese, *Phys. Rev.* **D53**, 5541 (1996);  
<https://doi.org/10.1103/PhysRevD.53.5541>
- [5] T. Damour and K. Nordvedt, Jr., *Phys. Rev.* **D48**, 3436, (1993);  
<https://doi.org/10.1103/PhysRevD.48.3436>
- [6] B. Bertotti and N. Ashby, *Classical and Quantum Gravity* **27**, (2010) 145013;  
<https://doi.org/10.1088/0264-9381/27/14/145013>
- [7]. K. S. Thorne, *Gravitational Radiation*, Ch. 9 in “300 Years of Gravitation,” eds S. W. Hawking and W. Israel, Cambridge University Press (1989), pp 330-458.
- [8] E. Peik *et al.* *J. Phys. B* 39 (2006) pp 145-158;  
<https://doi.org/10.1088/0953-4075/39/1/012>
- [9] M. Armano *et al.* *Phys. Rev. Letts.* **116**, (2016), 231101.  
<https://doi.org/10.1103/PhysRevLett.116.231101>