Topical: Rydberg atoms for fundamental physics in space

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1. Objective and Team

In a collaborative effort between several research groups, it is proposed to (1) design, build and apply instrumentation to measure the Rydberg constant based on spectroscopy of circular-state Rydberg atoms, and (2) to design and build instrumentation for cavity-QED-style detectors for single microwave photons, and to apply those towards an axion dark matter search. The project components are synergistic because circular Rydberg atoms of the same atomic species, Rb-87, are playing a central role in both of them. The preparation and circularization of Rydberg atoms in a space environment presents an important infrastructure-development component of the project. Here, we have assembled a research team with a wide range of expertise to bring the proposed research to fruition. The collaboration is formed by Dr. G. Raithel (experiment, University of Michigan), Dr. R. Thompson (experiment, JPL), Dr. V. S. Malinovsky (theory, ARL), Dr. A. Derevianko (theory, University of Nevada), and Dr. D. A. Anderson (experiment, Rydberg Technologies Inc., Ann Arbor). Dr. Raithel is an expert in Rydberg-atom spectroscopy, cavity-QED with Rydberg atoms, circular states, various types of atom traps, and interacting many-body systems. Dr. Thompson is experienced in atom cooling and trapping and cavity-QED. He also brings crucial experience in atomic-physics experiments on NASA's Cold Atom Lab (CAL). Dr. Anderson is an expert on Rydberg atoms, circular states, electromagnetic field sensing, and integration of complex laser apparatus. Dr. Malinovsky is an expert in quantum control theory and its applications to Rydberg atoms and optical lattices. Dr. Derevianko is an expert in fundamental atomic physics, precision metrology, and dark matter.

2. Background

Rydberg atoms [1] are an active field of modern physics with applications in precision measurements [2–5], molecular physics [6, 7], quantum control [8, 9], field sensing [10–13], nonlinear quantum optics [14–16], and as a platform for quantum computing and simulations [17-19]. Rydberg atoms have a long tradition in fundamental research, precision metrology, and fundamental constants. This includes cavity-QED experiments on the Jaynes-Cummings model [20, 21], quantum-non-demolition measurement [22], sub-Poissonian [23] and trapped quantum states of radiation [24, 25], and Schrödinger-cat states of microwave fields [26, 27]. These investigations have culminated in Professor Haroche's Nobel Prize in 2012. Recently-emerging quantum technologies [28] that utilize Rydberg atoms are, in part, build upon advances in electromagnetically induced transparency of Rydberg-atom vapors. Rydberg-atom-based field sensors exploit the legendary sensitivity of Rydberg atoms to microwave fields, reported among the first by Haroche, Walther *et al.* [29-31].

Contemporary work on Rydberg atoms is focused on the aforementioned applications in quantum field sensing, in large parts in vapor cells, on quantum simulation, in large parts in cold-atom systems, and on quantum control. The latter includes modern protocols for Rydberg atom circularization and optimal control [9, 32, 33]. Circular-state Rydberg atoms (CS), which are at the center of this whitepaper, are atoms in maximum angular momentum states. These states have long radiative lifetimes, only a small number of strong electromagnetic transitions, and low electric and magnetic susceptibilities when properly prepared in weak, static stabilization electric and magnetic fields. These properties make CS ideal for high-precision spectroscopy, field sensing, and the study of coherent quantum phenomena in the widest sense (quantum simulation, many-body physics, cavity-QED and so on). By extension of early work in [29-31], CS are also perfect candidates for the detection of single-photon microwave fields emitted by weak natural or man-

made RF radiation sources, a capability that is leveraged in the proposed search for axion dark matter (DM) in the 100-GHz range.

3. Proposed work

3.1 Rydberg atom spectroscopy in microgravity. From the properties of CS Rydberg atoms, several types of spectroscopic applications with different focal points of interest can be derived. In one line of work, the objective of high-precision spectroscopy is to explore fundamental properties of the atoms themselves as the center piece of study. This work includes a precision measurement of the Rydberg constant using CS atoms [4, 34-36], and can be extended to include a range of other atomic quantities, such as energy level splittings (fine, hyperfine etc.), atom-atom interactions in free space or in cavities that amplify coupling strengths, atomic and molecular behavior in external fields, and so forth.

In context with the proposed precision measurement of the Rydberg constant using CS Rydberg atoms, the large amount of activity and publicity around what is known as the "proton radius puzzle" is particularly noteworthy. The works [37-43] cited in this paragraph are NOT on Rydberg atoms, but on low-lying atomic states in which the Coulomb coupling in the atom, quantified via the Rydberg constant, and QED, quantified via Lamb shifts, and the proton size, quantified via an energy level shift proportional to the nuclear RMS charge radius, are all very important. Experimentally obtained data require reconciliation of these effects to extract a specific quantity (the nuclear radius, for instance). The proton-radius controversy began in 2010 with a measurement of a smaller-than-expected value in muonic hydrogen [37, 38]. This was reviewed in 2015 in context with nuclear scattering data [39]. Measurements on electronic hydrogen were giving various results (for one of many, see [40]). The latest [41] is agreement with the smaller proton radius. Electron scattering data [42, 43] also congregate towards the smaller proton radius.

In this whitepaper, the first experimental plan is to follow up on the cited ideas to measure the Rydberg constant with CS Rydberg atoms. The proposed method removes the interconnection between Coulomb, QED and nuclear-charge-overlap effects in the spectroscopic measurements by virtue of the semi-classical, mesoscopic nature of CS Rydberg atoms. CS atoms have negligible QED and nuclear-overlap-induced shifts, allowing one to back out the Rydberg constant from spectroscopic data without these interconnections. The obtained "clean" measurement of the Rydberg constant will present a value based upon an entirely independent method. Once obtained, this value can be compared with the current CODATA value, thereby also providing a contribution to addressing the proton radius puzzle. To achieve this feat, it will be necessary to measure several CS Rydberg resonances in the 100-GHz range to near 0.1 Hz systematic and statistical uncertainty in order to reach the CODATA relative uncertainty for the Rydberg constant, which stands at 1.9×10^{-12}. The associated challenges have been discussed in detail in [4].

3.2 Single-microwave-photon detectors. A second line of work is the application of Rydberg atoms, and in particular CS Rydberg atoms, for the measurement of weak radiation sources. In recent years there has been a wide range of activity in using Rydberg-EIT and microwave-to-light conversion in vapor cells or vacuum systems to achieve an all-optical measurement of a radio-frequency field (some of this research is reviewed in [28]). In context with applications in space, we see potential for Rydberg-atom-based field sensors for advanced communications, remote sensing, precision microwave radiometry, and the detection of black-body radiation (CMB, planetary surfaces, astrophysical sources etc.).

There also has been considerable interest in the conversion of a single RF photon into a state change of a particle, where (1) the conversion cross section is large enough to guarantee the

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conversion of the photon, (2) the matter particle *and* its internal quantum state are long-lived enough so they can be detected with near-100% efficiency, and (3) the efficiency and the selectivity of the detector to read the converted state are near-100%. The approach amounts to capturing and storing the photon in a microwave resonator, using a resonant, strong Rydberg dipole transition to effect a Rydberg-state change, and to measure the changed state of the Rydberg atom with a single-particle state-selective detector. There are efforts underway [44-46] to use Rydberg atoms to perform exactly this function in order to detect single photons emanating from the conversion of a dark-matter (DM) axion in a process that delivers a photon. That single RF photon is temporarily held, or shelved, in a high-Q microwave cavity for eventual pick-up by an atom. Technically, the involved atom-field physics is related to work on Rydberg-atom cavity QED mentioned in the Section 2. The Primakoff conversion process that creates the photon occurs in a cavity that is held in a magnetic field [44-46]. The photon is transmitted into a sensing cavity in which the Rydberg atom changes state upon conversion of the photon into a quantum-state change [47]. In this way, low-energy DM in the range between 1 GHz and 100 GHz can be detected. Owing to the immense importance of any DM detections, the expected rarity of the Primakoff conversion events has not prevented agencies from investing into this type of search [44-46]. At the core of the approach lies a single-microwave-photon detector based on Rydberg atoms.

4. Microgravity relevance

4.1 Why Rydberg-atom spectroscopy in space. The long lifetime of CS, which can reach 100's of milliseconds, presents a main reason why CS are a particularly worthwhile type of Rydberg atoms to research on in space. While the drop distance of an ultra-cold atom on Earth over a few 100 milliseconds is only 10's of centimeters, this drop distance is too large to allow for sufficient elimination of systematic shifts from external electric and magnetic stray fields [4]. The atoms must be held in place while they are being probed spectroscopically, either by an atom trap or by virtue of being in microgravity.

We first consider the use of an atom trap to levitate the CS atoms against gravity on Earth. Atom-trap-induced shifts present the leading systematic in case studies of high-precision spectroscopy of CS atoms [4]. At the same time, the trap-induced shift must be reduced to ~0.1 Hz. We do not see a way to achieve this in a 1-g environment with any type of atom trap or levitation device.

Further, in a scheme that we have proposed in [4] and analyzed further in [48], a microwave-modulated optical-lattice trap is applied to the atoms for the purpose of probing an optical CS quadrupole transition that is free of both linear Stark and linear Zeeman shifts, eliminating other sources of damaging systematics. If deployed, the modulated optical lattice must be an extremely shallow one (depth in the sub-100-Hz range) in order to comply with the limit of ~0.1 Hz on the trap-induced shift. Gravity on Earth does not allow for such shallow optical-lattice traps.

In a microgravity environment, there is no need to levitate the CS atoms against gravity, there is no strict requirement to confine the CS atoms, and any applied optical lattices can be made near-arbitrarily shallow. Hence, microgravity is required to reduce residual trap-induced shifts in the proposed study to below ~ 0.1 Hz, and to thereby allow the proposed Rydberg-constant measurement to become competitive. A space-based platform, or a terrestrial installation providing the required degree and duration of microgravity, will provide that essential requirement.

4.2 Why single-photon microwave detectors in space. In the proposed detector to measure the Primakoff conversion of axions into microwave photons, the converted photons are shelved in a high-Q microwave resonator, which holds them for guaranteed pickup by a CS Rydberg atom. CS atoms are shuttled on an optical conveyor belt through the pickup cavity, in which they are

transitioned via rapid adiabatic passage or STIRAP [49] from one CS Rydberg level into another, in the case that a converted photon is present. If there is no photon, there is no transition.

Based upon the lifetime of the CS atoms of about 100 ms, we may estimate conveyor speed, cavity size, cavity Q-value, detector bandwidth, single-photon Rabi frequency in the cavity, and the maximum conveyor-induced level shift the CS may have while they interact with the converted photon. This type of estimation leads to an allowed background acceleration acting upon the CS atoms traveling in the conveyor in the range of 10^{-4} to 10^{-5} g. This means the proposed method of axion DM detection will work best in microgravity. Further, as continuous operation will be desired in view of the grand goal, a semi-permanent deployment in space may be most ideal.

An additional reason for single-photon microwave detectors in space is to make them available for the detection and analysis of weak astrophysical RF sources without terrestrial electromagnetic interference. This includes the CMB, which falls within the RF detection range of Rydberg atoms (the latter ranges from sub-GHz to sub-THz). Rydberg-atom microwave sensors could become a secondary CMB detection platform, after the widely used transition-edge sensors. Further, Rydberg-atom-based sensors can be employed in thermal imaging of celestial objects, including measurements of radiation from planets and asteroids, from CO molecules in interstellar clouds, or from molecular or Rydberg-atom masers in deep space.

5 Beneficiaries

The proposed work will benefit the science community by providing measurements for fundamental constants such as the Rydberg constant. The Rydberg constant is an important node in the system of fundamental constants (available at CODATA [50]). Accurate and precise values for the constants reflect the accuracy of our current understanding of the physical world, and are therefore of fundamental importance. The same applies to all ongoing DM searches. The proposed space-based detector for single microwave photons may benefit the search for axion DM in the 100-GHz range. The single-photon-detection work may benefit other areas in fundamental and applied science (see last paragraph in Section 4).

A collateral benefit relies in the use of Rydberg atoms in quantum information, communication and cryptography. While the goals are different, the technology on how to prepare and handle Rydberg atoms is shared, in part, between the fields of use. Quantum information, communication and cryptography are beneficial in terrestrial applications as well as in complementary applications in space (such as specialized communications satellites and base stations in space).

6. Selected components of the approach

6.1 Rydberg constant. In the left of the first figure, we show CS Rydberg atoms prepared in static stabilization fields (F, B). The Rydberg constant is extracted from measurements of CS transitions of the type $|n, n_1 = 0, n_2 = 0\rangle \rightarrow |n+2, n_1 = 1, n_2 = 1\rangle$, written in parabolic quantum numbers [1]. This transition is free of critical systematic shifts [4], but it requires a two-photon microwave drive or an all-optical drive via the ponderomotive effect in amplitude-modulated optical lattices. In the right of the first figure, we visualize the concept of a clean measurement of the Rydberg constant by removing QED effects and the effects of the nuclear charge radius from the measurement.

It is proposed to proceed in four stages. The first is to prepare Rydberg atoms, electron and ion detectors in a space-based vacuum system modeled after the Cold Atom Lab (CAL), leveraging JPL's leading function at CAL. We will use compact laser sources for Rydberg-atom preparation,
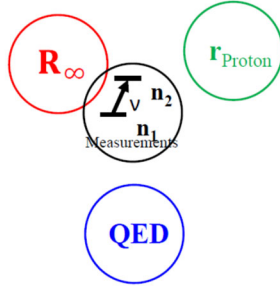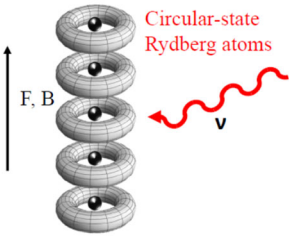
to be acquired, developed and packaged by the industry partner involved in the work. Laser-beam handling and routing will build upon JPL's expertise at CAL and expertise of the involved academic and industrial partners. In the second stage, Rydberg-atom circularization using optimal control and rapid adiabatic passage will be modeled by partners from atomic theory, who will advise partners on the experiment with practical implementation. In the third stage, CS spectroscopy will be conducted using the two-photon microwave drive, which has favorable microwave AC shifts. The data reconciliation will require a concerted effort of partners, which will yield a preliminary result in the Rydberg constant measurement effort. In the fourth stage, an amplitude-modulated lattice will be deployed to eliminate microwave AC shifts from the spectra, leading to a Rydberg-constant value that is free of fundamental QED and nuclear-overlap perturbations, free of microwave shifts, and with remaining systematics reconciled.

6.2. DM search. The proposed DM search, sketched in the second figure, will also draw from the combined expertise of the involved academic, industrial and JPL partners. The cavity-QED system consists of
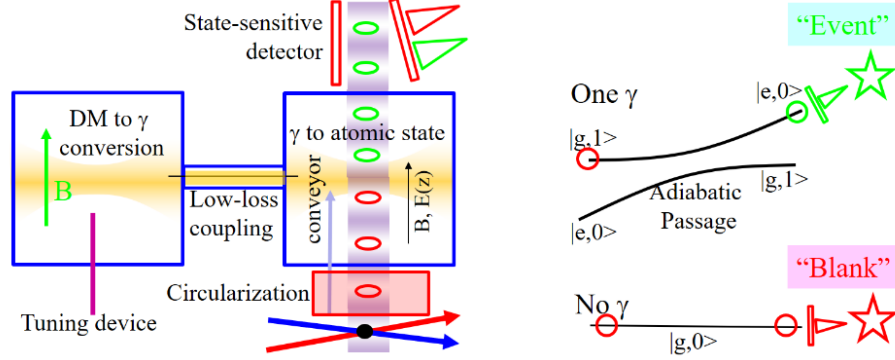
two resonantly coupled cavities and a resonant atom. The targeted Q-value of the cavities is about 10^8. The DM-to-γ conversion occurs in the left cavity, which harbors a magnetic field and can be tuned relative to the second. One resonant Rydberg atom at a time is dragged through the field mode of the right cavity (orange) by an optical conveyor (purple) that moves at ~1m/s speed. Atoms are laser-excited (red and blue arrows), circularized (red box), and state-selectively detected after the cavity. A cavity-internal, slightly z-dependent stabilization field adds a z-dependent detuning, which is fashioned such that the atoms undergo an adiabatic passage from Rydberg state |g> into Rydberg state |e> if there is a photon present, otherwise the atom remains in |g>. Given a sufficiently large single-photon Rabi frequency of the transition, the adiabatic passage is near-100% efficient. The process transfers the field state into a unique atomic state for delayed, efficient and robust state-selective readout. Most positive events will be due to thermal photons, whose number and production rate must be kept low through the use of cryogenics. The injection of DM-converted photons is controlled via tuning of the left cavity, which allows one to enable or disable the recording of DM events.

The DM search amounts to a search for a small fraction of unexplained microwave photon counts. The theory component will include a determination of the background rate of black-body-induced events to set the signal-to-noise for the DM-induced events. Design parameters include the conveyor depth and atom-to-atom period, for which ~ 200 Hz and 1 cm appear suitable at the time of preliminary planning. It is a part of the proposed work to optimize these design parameters, to fabricate prototypes for ground tests, and to develop blueprints for a dedicated effort to find DM.

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