Research Campaign White Paper: Satellite Quantum Test of the Universality of Free Fall

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Abstract: Quantum sensors based on the interference of matter waves provide an exceptional access to test the postulates of General Relativity by comparing the free-fall acceleration of matter waves of different composition. We present the scientific motivation for a satellite quantum test of the universality of free fall at the 10^{-17} level or better beyond current limits set by classical experiments.

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1 Introduction

Einstein's theory of general relativity (GR) is a cornerstone of our current description of the physical world. It is used to understand the flow of time in the presence of gravity, the motion of bodies from satellites to galaxy clusters, the propagation of electromagnetic waves in the presence of massive bodies, the generation and propagation of gravitational waves, the evolution of stars, and of the Universe as a whole. There is however a strong asymmetry between GR and the other interactions of the standard model of particle physics (SM): the electromagnetic, weak, and strong interactions. Whilst these latter couple to some specific property or charge, gravitation is universally coupled, meaning that it couples in the same way to any mass/energy, which allows a geometric description of gravitation as the effect of the curvature of space-time. The phenomenological manifestation of this universal coupling is known as the Einstein Equivalence Principle (EEP), and is central to modern physics at all scales [1, 2, 3].

The EEP is not a fundamental symmetry, like *e.g.* gauge invariance in the SM, but rather an experimental fact. Einstein himself initially called it the *hypothesis of equivalence* before elevating it to a *principle* once it became clear how central it was in the generalization of special relativity to include gravitation. And indeed, from a SM perspective it is rather surprising that the EEP should be satisfied at all, let alone at the stringent uncertainties of present-day tests. Furthermore, the difficulties in quantizing GR and in unifying it with the SM give further indications that the EEP could be violated at some level. For example, most attempts at unification theories involve additional fields, that have no good reason to couple universally to the SM and thus would violate the EEP. Similarly, the unknown nature of dark energy and dark matter postulated by modern cosmology and astronomy, is often "explained" by invoking additional fields that permeate spacetime. Again such fields would in general couple non-universally to the SM and thus violate the EEP.

These considerations, detailed in reference [4] and subject of this decadal's white paper "To-wards gravity's frontiers", make experimental tests of the EEP one of the most promising roads to discovering new physics beyond the SM and GR. By doing so one may shed new light on much of our present day understanding of the universe, and in particular its main constituents, cold dark matter and dark energy, both of which we know nothing about apart from their gravitational manifestations. Additionally, diversifying the tests by using new forms of test-masses e.g. atoms in quantum superpositions, may give access to the interplay between the SM and GR at the most fundamental level.

Exploring the extent to which the EEP is satisfied is then the main subject of this white paper largely following the scenario of reference [5] proposed in the frame of ESA's Voyage 2050 call for ideas. Finding a violation of the EEP would not only revolutionise physics as a whole, but certainly also shed new light on astrophysics and cosmology, in particular concerning its dark components.

The history of experimental tests of the EEP dates back at least as far as the 16th century and Galileo Galilei. Since then, tremendous efforts have been carried out to push laboratory tests to uncertainties as low as parts in 10^{-12} in quantum tests of the universality of free fall (UFF) [6] and 10^{-13} for the classical counterparts ¹. However, ground tests are ultimately limited by the Earth's gravitational environment, and future discoveries will come from space experiments, like

¹Tests of UFF are generally quantified by the Eötvös ratio defined as $\eta = 2(a_1 - a_2)/(a_1 + a_2)$ where $a_{1,2}$ are the gravitational accelerations of two test masses of different compositions.

the recent MICROSCOPE experiment, which between 2016 and 2018 tested the UFF in space. First partial results excluded a violation of the EEP at the 10^{-14} level [7] whilst final results will search for a violation down to the low 10^{-15} region [8].

Over the last years, many proposals for space-tests of UFF have been put forward, e.g. STEP [9], GG [10], POEM [11], GAUGE [12], STE-QUEST [13, 4], and the future will certainly be built on these and the heritage of MICROSCOPE.

In the post-MICROSCOPE era, the subject of this white paper, the aim will be to discover a UFF violation at the 10^{-17} level, a leap in sensitivity by more than two orders of magnitude. A strong technology development has been ongoing over the last years, in particular in the context of cold atom interferometry in microgravity through the QUANTUS, MAIUS and BECCAL projects [14, 15, 16, 17] in Germany, the ICE project in France [18, 19] and the CAL project in the US [20]. Additionally, recent theoretical and experimental results have allowed to strongly reduce some of the main systematic effects in these experiments such as by employing gravity gradient cancellation (GGC) techniques [21, 22, 23, 24]. The mission concept presented here is based on cold-atom technology following the STE-QUEST proposal and features low Earth orbits with drag-free technology as convincingly demonstrated by MICROSCOPE and LISA-Pathfinder [26, 27].

2 Scientific motivation

Most attempts at quantum gravity and unification theories lead to a violation of the EEP [28, 29, 30, 31, 32, 33], which in general have to be handled by some tuning mechanism in order to make the theory compatible with existing limits on EEP violation. For example, in string theory moduli fields need to be rendered massive (short range) [28] or stabilized by *e.g.* cosmological considerations [29] in order to avoid the stringent limits already imposed by EEP tests. Similarly M-theory and Brane-world scenarios using large or compactified extra dimensions need some mechanism to avoid existing experimental limits from EEP tests or tests of the inverse square law [31, 33, 32, 34, 35]. Therefore, not only do we expect a violation of EEP at some level, but the non-observation of such a violation with improving uncertainty is already one of the major experimental constraints for the development of new theories in the quest for quantum gravity and unification. This makes experimental tests of EEP one of the most essential enterprises of fundamental physics today.

It is interesting to note that experimental constraints for EEP violations at low energy are rather closely related to present-day physics at the very small scale (particle physics) and the very large scale (cosmology). Notably, the experimental confirmation of the Higgs boson has thus lent strong credibility to the existence of scalar fields, as the Higgs is the first fundamental scalar field observed in Nature. It is thus likely that additional long and/or short range scalar fields exist, as postulated by many unification theories, and EEP tests are one of the most promising experimental means for their observation. At the other extreme, if such scalar fields are massive they may well constitute the mysterious dark matter (DM) of cosmology. There is no reason for such DM to be universally coupled to SM fields, and it would thus give rise to a violation of the EEP that could be detected by EEP tests [36]. Additionally, most models for Dark Energy (DE) are also based on long-range scalar fields that, when considered in the context of particle physics, are non-universally coupled to the fields of the SM [37, 38]. Similar reasoning applies to spin-1 bosonic fields that also may violate the EEP [39, 40]. As a consequence, one would expect EEP violations from such fields, be

it DM and/or DE at some level, which might be observed with low energy experiments, like the ones discussed here. Such a detection would provide a very appealing route towards independent confirmation of DM/DE, making it more tangible than only a hypothesis for otherwise unexplained astronomical observations.

3 Dual atom-interferometric test of the UFF in space

The coherent manipulation of cold atoms with electromagnetic fields is key to new types of sensors with various metrological applications. Indeed, time and frequency are today's best realized physical units, thanks to atomic clocks based on optical and microwave transitions. Moreover, freely falling atoms constitute excellent test masses, hence allowing to infer inertial quantities through interferometric measurements. In particular, their long-term stability and high accuracy render atomic gyroscopes [41] and accelerometers [42] exquisite tool for navigational, geodesic and fundamental [43, 44, 45, 23, 46, 47] applications.

A concurrent operation of two such accelerometers with different atomic species provides a new pathway to tests of the UFF. Experiments based on this concept significantly extend the range of test pairs so as to include previously inaccessible species and hence prove invaluable to explore many facets of different violation scenarios such as the Standard Model Extension [48]. Moreover, phenomena exclusive to quantum systems, such as coupling of gravity to spin [49], or superposition of electronic states [44], and adopting entangled atoms of different species as test masses [50] provide a unique insight into the interface of gravity and quantum mechanics.

Atom interferometry exploits the wave nature of matter to infer metrological quantities through interference. As an example, the source of such an interferometer could be a dual species Bose-Einstein Condensate of ⁸⁷Rb and ⁴¹K starting from an on-chip 3D magneto-optical trap loaded by a cold beam from a 2D magneto-optical trap, molasses cooled, evaporated inside in a magnetic chip trap and finally transferred to and condensed inside an optical dipole trap. After dropping or launching these matter waves in free fall, they are subject to a series of light pulses, which serve as beam splitters and mirrors in close analogy to optical Mach-Zehnder interferometers. Through a stimulated two-photon process, such a light pulse transfers momentum to an atom and imprints a position dependent phase. A first beam splitter puts the atoms into a superposition of two motional states, which travel along different trajectories before being redirected by a mirror pulse and finally recombined by another beam splitter. The two output ports of the interferometer differ in momentum, and their relative population depends on the phase difference accumulated between the two branches. Since at each light pulse, the position of the atoms is referenced to the light field, this phase difference is indicative of the free fall acceleration a of the matter waves with respect to the apparatus. To first order, the phase is $\Delta \phi = KaT^2$, where K is the effective wave number quantifying the momentum transferred at each pulse and T is the pulse separation time. In a differential measurement with two species A and B, the differential acceleration uncertainty per experimental cycle

$$\sigma_{\Delta a} = \left[\left(\frac{1}{C_A K_A T_A^2 \sqrt{N_A}} \right)^2 + \left(\frac{1}{C_B K_B T_B^2 \sqrt{N_B}} \right)^2 \right]^{1/2} \tag{1}$$

is limited by the quantum-projection noise (shot noise), given by the number N of atoms contributing to the signal. The contrast C accounts for the visibility of the interference fringes. Typically,

a retro-reflective setup is employed, such that the same mirror serves as a reference for both interferometers, which are operated simultaneously. This leads to common mode rejection for various systematics and noise sources, where the suppression factor depends on the choice of the atomic species.

Ultimately, such an experiment would monitor the motion of two atomic wave packets with initially superposed centers. It can be interpreted as a test of classical general relativity coupled to a Klein-Gordon field in a non-relativistic limit or, equivalently, a Schrödinger equation with an external gravitational potential. The sensitivity to violations of the UFF is quantified by the Eötvös ratio $\sigma_{\Delta a}/g$ and suggests operation on a low-Earth orbit.

Parameters	
Atom number N	10^{6}
Effective wave number <i>K</i>	
Rb	$8\pi/(780nm)$
K	$8\pi/(767nm)$
Free evolution time <i>T</i>	20 <i>s</i>
Cycle time T_c	10 <i>s</i>
Simultaneous interferometers	5
Contrast C	1
Orbit	
Semi major axis h	700km
Ellipticity e	$\leq 10^{-4}$
Single shot diff. acc. sensitivity Integration over one orbit Integration time to $\delta \eta = 10^{-17}$	$1.09 \times 10^{-13} \text{ m/s}^2$ $8.8 \times 10^{-16} \text{ m/s}^2$ 18 months

Table 1: Parameters for a quantum test of the UFF targeting $\delta\eta \leqslant 10^{-17}$. Table taken from reference [5] under the Creative Commons Attribution 4.0 International license http://creativecommons.org/licenses/by/4.0/

3.1 Operation mode

The atom shot-noise limited uncertainty in the Eötvös ratio displays the maximal achievable sensitivity to a potential violation signal for such sensors, assuming that systematic and stochastic errors can be kept below this level. Since atom shot noise is uncorrelated from shot to shot, it may be averaged down over many repeated cycles. In the following, we will consider a space-borne mission on a circular orbit, where the satellite is kept inertial with respect to distant stars. For the determination of the Eötvös ratio, the integration of the signal needs to take into account the varying projection of the gravitational acceleration g onto the sensitive axis [22], such that the averaging over n measurements reads

$$\sigma_{\eta} = \frac{1}{\sqrt{n}} \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} \left(\frac{\sigma_{\Delta a}}{g(t_j)}\right)^2}.$$
 (2)

The number of beneficial measurements per orbit is reduced by the local projection of g becoming too small for certain orbital positions. For an inertial satellite on a circular orbit with orbital frequency Ω , it can be written as $g(t_i) = g_0 \cos(j\Omega T_c)$.

Aiming for a target uncertainty of $\sigma_{\eta} \leq 10^{-17}$ suggests parameters as presented in Table 1. We assume a moderate beam splitting order of 2 in order to keep the spatial extent of the interferometers below one meter. Moreover, we suppose typical atomic numbers and cycle time for the generation and engineering of BECs. Assuming that 10s are required for the atomic source preparation, followed by an interferometer of 2T=40s duration, the stated cycle time requires an interleaved operation of 5 concurrent interferometers. The contrast can be assumed to be near unity, since major sources of contrast loss, such as gravity gradients, can be mitigated as outlined in [22]. Given an altitude of $h=700 {\rm Km}$ and a cycle time of $T_c=10s$, a maximum of 356 measurements per orbit allows to integrate the shot-noise limited Eötvös ratio to 8.8×10^{-16} after one orbit, such that a total of $\tau=18 {\rm months}$ of integration are required to reach $\sigma_{\eta} \leq 10^{-17}$ as shown in figure 1.

3.2 Mission requirements

Any spurious differential acceleration between the two species can, a priori, not be distinguished from a potential UFF violation signal. Consequently, random acceleration contributions need to be kept below the atomic shot-noise. All systematic error sources have to be controlled at a level better than the target inaccuracy of 10^{-17} , or be modulated at other frequencies than the local projection of g. In general, one can decompose the differential acceleration into its frequency components,

$$\Delta a = \delta a \cos(\omega_0 t) + \Delta a_{\text{const}} + \sum_{j=0} \Delta a_{\text{sys}}^j \cos(\omega_j t), \tag{3}$$

where δa is the potential violation signal that is to be detected, $\Delta a_{\rm sys}^j$ a systematic acceleration contribution at frequency ω_j and $\Delta a_{\rm const}$ comprises all non-modulated terms [51]. Demodulation of the signal frequency ω_0 , at which a possible violation signal is expected, averages all other frequency components down,

$$\frac{2}{\tau} \int_0^{\tau} \Delta a \cos(\omega_0 t) dt \le \left(\delta a + \Delta a_{\text{sys}}^0\right) + \frac{2}{\tau \omega_0} \left(\frac{\delta a}{2} + |\Delta a_{\text{const}}| + \frac{4}{3} \sum_{j=1} \left|\Delta a_{\text{sys}}^{(j)}\right|\right),\tag{4}$$

where τ is the duration of integration [22]. This is a pessimistic upper bound, since for appropriate choices of τ , the integral over certain frequency components is trivial. The key insight is that the violation signal is demodulated to DC, while all systematic contributions are averaged down at a rate inversely proportional to ω_0 . This fact is, for example, employed in MICROSCOPE [7], where the satellite is additionally spun for an improved integration rate. We, however, consider a mission in which the satellite is kept inertial with respect to distant stars, such that ω_0 corresponds to the orbital frequency. Obviously, differential acceleration contributions $\Delta a_{\rm sys}^0$ modulated at ω_0 can not be discriminated from a potential violation signal with this technique, and therefore have to be well-controlled.

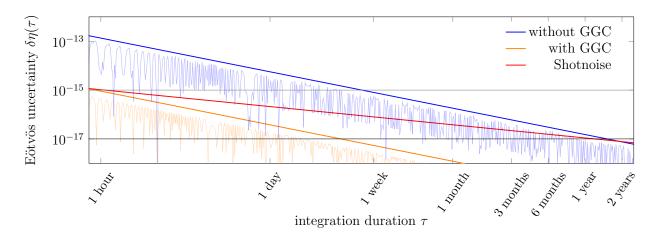


Figure 1: Integration of systematic uncertainties due to gravity gradients in a UFF test with Rb and K. GGC significantly reduces the systematic contributions, such that the residual differential acceleration may be attenuated to unprecedented degree through signal demodulation (orange curve). This does not only allow for largely reduced requirements on the source preparation for mission proposals as STE-QUEST but also paves the way for more ambitious mission scenarios targeting $\delta \eta \leq 10^{-17}$ in shot-noise limited operation (red curve). In comparison, although the systematics are integrated down thanks to demodulation, the measurement would be limited by systematics without GGC (blue curve). Figure taken from reference [22] under the Creative Commons Attribution 4.0 International license http://creativecommons.org/licenses/by/4.0/.

4 Conclusion and recommendation

A dual species atom interferometer in space offers a new approach for testing the universality of free fall, complementary to classical tests. The described scenario builds on heritage which demonstrated atom optics and atom interferometry on microgravity platforms, such as parabolic flights [18, 19], drop tower [14], sounding rockets [15, 16], aboard the International Space Station [20] and will utilize these same platforms in the near-term to validate that all possible systematics will be sufficiently suppressed via environmental control or signal demodulation. It anticipates a residual uncertainty in the Eötvös ratio of 10^{-17} after 18 months of integration as detailed in references [5] and [22]. The maturation of hardware and interferometry techniques' TRL required by this mission concept could be reached taking advantage of the CAL and BEC-CAL facilities, in a dedicated demonstrator in the Einstein Elevator in Hanover, Germany (see topical white paper "Earth-based platforms for microgravity research on ultra-cold atom devices for space applications") or aboard a space station (see the Research Campaign paper "Quantum Test of the Universality of Free Fall in Earth's Orbit"). Beside an investment in dedicated demonstrators (elevator and space station), a participation to an STE-QUEST-like medium-size mission (cost at Completion of 550 M€) within ESA's science program (launch around 2037) could be an interesting opportunity. The call for such a mission has been published in December 2021 (https://www.cosmos.esa.int/web/call-for-missions-2021).

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