FUNDAMENTAL PHYSICS AND LUNAR SCIENCE INVESTIGATIONS WITH ADVANCED LUNAR LASER RANGING

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Executive Summary: Space offers unique experimental conditions and a wide range of opportunities to explore the foundations of modern physics with an accuracy far beyond that of ground-based experiments. Space-based experiments today can uniquely address important questions related to the fundamental laws of Nature. In that respect, lunar laser ranging (LLR) has made significant contributions to fundamental physics research as well as to our understanding of the moon's internal structure and the dynamics of the Earth-moon system. In fundamental and gravitational physics, LLR is used to perform high-accuracy tests of the Equivalence Principle, to search for a time-variation in the gravitational constant, and to test alternative theories of gravity. The LLR data provide insight over broad areas spanning lunar and Earth sciences, geodesy and geodynamics, solar system ephemerides, gravitational physics, terrestrial and celestial reference frames. LLR expands our understanding of the motion of the Earth's axis in space including its precession and nutation, Earth's orientation and its obliquity to the ecliptic, the intersection of the celestial equator and the ecliptic, lunar and solar solid body tides, lunar tidal deceleration, lunar physical and free librations, the structure of the moon and energy dissipation in the lunar interior, and the study of core effects.

LLR measurement accuracy today is limited by the corner cube retroreflector (CCR) instruments currently on the lunar surface which are arrays of small CCRs (Figure 1). A laser is aimed at each of these CCR units, returns the beam parallel to itself. Since range measurements are based on single-photon timing events there is an ambiguity in knowing exactly which CCR reflected the photon. LLR science is also limited by the lack of CCRs in the southern hemisphere, and by increased difficulty in performing ranging due to gradual degradation in the CCRs deployed on the moon 50+ years ago.

Deployments of several new CCR instruments on the moon may occur in 2022-27 by the NASA Artemis program and related commercial opportunities due to increased interest in lunar network science, with LLR providing information complementary to seismic and heat-flow instruments, with improved geographic distribution. This enhancement of the LLR infrastructure will lead to major progress in both fundamental physics and lunar science research. As a result, the BPS Decadal has an unprecedented role in enabling impressive progress in multiple areas of science. Our goal is to expand NASA's science objectives in space by enhancing "fundamental physics" as an important element in agency's ongoing space research and exploration efforts.

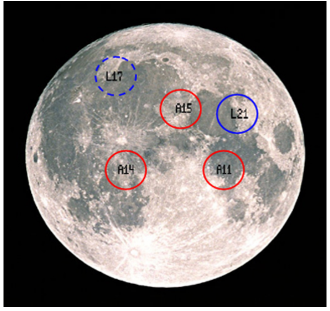

Figure 1. LLR lunar retroreflector sites, operational after 50+ years.

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1. The Living Legacy of Apollo Program

An LLR experiment involves three major components: i) the design/deployment on the moon of CCR instruments, ii) the design/operation of Earth-based ranging stations to support LLR operations, and iii) modeling efforts, LLR data analysis, derivation of the resulting science products.

The first deployment of a CCR array on the lunar surface took place during the Apollo 11 mission (Figure 2) in 1969, making LLR a reality (Bender et al., 1973). The goal was to place arrays at three lunar locations to study the moon's motion (NASA, 1971). Two more CCR packages were set up by the Apollo 14 and 15 astronauts. In addition, two French-built CCR arrays were on the Lunokhod 1 and 2 rovers placed on the moon by the Soviet Luna 17 (1970), and Luna 21 (1973) missions, respectively (Figure 3). Figure 1 shows the five LLR reflector sites on the moon.

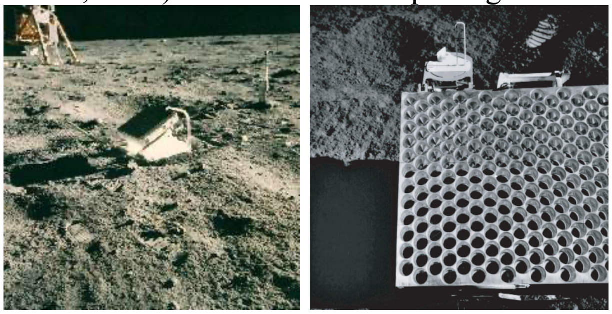

Figure 2. Apollo 11 (left) and (right) Apollo 15 laser CCR arrays on the lunar surface.

Most LLR scientific results come from long observing campaigns at several observatories. One station is the McDonald Laser Ranging System (MLRS) in Texas, USA (Shelus et al., 2002, http://www.csr.utexas.edu/mlrs/). This 0.76 m diameter station ranged from its current site during 1988-2013. Earlier McDonald data is available from other McDonald sites back to 1969. Another station is at the Observatoire de la Côte d'Azur (OCA) in France (Veillet et al., 1993; Samain et al., 1998, http://www.obs-nice.fr/). OCA began accurate observations in 1984. Both stations operate in a multiple-target mode, observing artificial satellites in addition to the lunar CCR arrays.

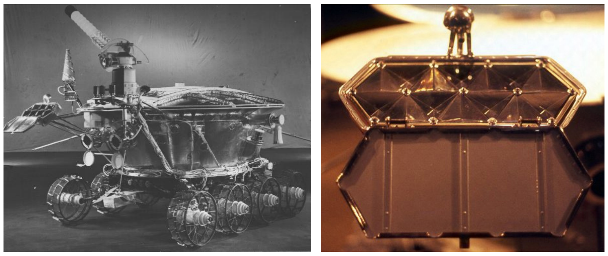

Figure 3. Lunokhod 1 with the retroreflector array sticking out at far left and a photo of its reflector.

The Apache Point Observatory Lunar Laser-ranging Operations (APOLLO) started ranging in 2006 (Murphy et al., 2013, http://www.physics.ucsd.edu/~tmurphy/apollo/apollo.html). With a 3.5 m diameter telescope, this facility is designed to achieve mm range precision and corresponding order-of-magnitude gains in the determination of relevant physics parameters (Murphy et al., 2002; Williams et al., 2004a). The APOLLO facility is funded jointly by NSF and NASA.

2. The opportunity

LLR is made by measuring the time between transmission of a laser pulse from an Earth tracking station to a CCR on the lunar surface and back to the tracking station. LLR measurement accuracy was originally limited by the length of a laser pulse generated for transmission to the moon. Advancements in laser technology make very short high-power laser pulses possible. LLR measurement accuracy now is limited *by the CCRs currently on the lunar surface*. These instruments are arrays of small CCRs and, since range measurements are based on single-photon timing events, there is an ambiguity in knowing which of the CCRs in the array the photon was reflected from.

This ambiguity results in an increase of the width of the returned pulses, thereby, contributing to a large measurement error. A fraction of this error can be statistically corrected by collecting many photons, but the remaining part is left in the data, limiting the progress in LLR science. The small CCRs used in the existing arrays have the advantages of intrinsically more thermal stability than larger CCRs, and of relatively large diffraction beam patterns, which allow for a return beam that is broad enough to account for the motion of the tracking station between the time a laser pulse is transmitted and the time the pulse returns to the tracking station 2.5 sec later. With their large target uncertainty and their aged performance, the current CCRs on the moon hindering the progress in LLR-enabled lunar science investigations. Therefore*, new CCRs on the moon are needed*!

We should consider placing on the moon new CCR instruments whose design is based on a single large CCR (Turyshev et al., 2013). The new instruments (i) have range uncertainty of <0.1 mm yielding *a factor of ~100 improvement in LLR measurement accuracy* for single photon, (ii) *have large enough signal* to be detected by many ground stations, and (iii) *have low mass* to be deployed robotically. The new CCR instruments overcomes several issues (beam pattern, thermal distortion, and velocity aberration) that discouraged the use of single CCRs previously but are solved now.

Recent progress in the development of high-power CW lasers for optical communication and laser ranging, enables major advances in LLR precision. The high power of these lasers allows for simultaneous range measurements to be taken between multiple lunar CCR arrays. Differencing these ranges could largely eliminate atmospheric perturbations that currently limit range accuracy (Svehla, 2015). This can be done by switching between two (or several) CCR arrays on the Moon. The resulting differential LLR architecture would have a significantly reduced atmospheric contribution and site-related errors improving the sensitivity needed to study lunar interior. Such precision range data would allow for determination and study of the lunar core size, shape, rotation, and turbulence, as well as interior rigidity and possible regions of partial melting.

Recognizing the value of the LLR for studies of the deep lunar interior, in 2016 JPL begun an upgrade at the JPL's Table Mountain Observatory (TMO), turning it into a unique LLR facility. The new LLR station uses a high-power CW laser (~1.1 kW, compare this to the current largest LLR laser of ~2W) bringing about major advances in the precision of LLR measurements (see Figure 4). Equipped with a high-power laser at $\lambda = 1064$ nm, TMO receives many more return photons (a flux of $\sim 10^4$ photons/sec) and obtains a much higher signal-to-noise ratio (SNR) than any other LLR facility; it enters a new photon-rich regime where differential LLR is feasible. In such a regime, (nearly-)simultaneous ranging to two or more CCR arrays is possible. With high photon flux, the fundamental limit to LLR accuracy is no longer sqrt(N), but Earth's atmosphere. This new LLR station may work even with smaller CCRs on the moon: the TMO's link budget allows LLR operations to a single CCR of 3.8 cm, reducing mass of the instrument, improving its landing robustness.

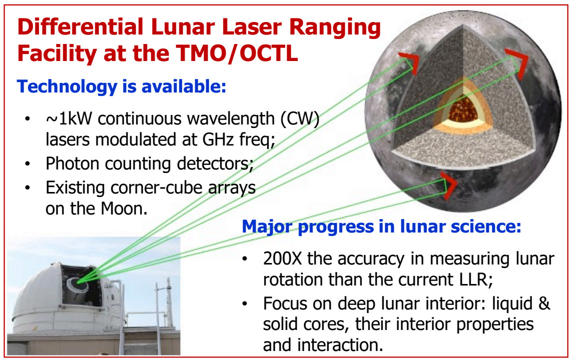

Figure 4. The schematics of the differential LLR.

The *differenced* delay error to any of the lunar CCRs can be as little as ~30 μm – a factor of ~200 better than is currently possible (Svehla et al. 2015). The new LLR facility will dramatically improve our knowledge of the lunar interior, beyond the contributions from the GRAIL mission and current LLR (Williams et al. 2014). Together with the new CCRs, the new LLR station will enable major progress in lunar science, which is the greatest motivation for new CCR instruments.

3. Expected Significance

The order-of-magnitude gain in the accuracy of weighted residuals achieved during ~50 years from the 1970's to the present (from ~25 cm to ~1 cm, respectively) has had a pronounced impact on many lines of LLR science investigations. The emerging LLR science opportunities may benefit from the new LLR station at the TMO. Advanced LLR campaign could enable 0.1-mm-level LLR ranging − a factor of 10 gain compared to the present state − and to widening the array distribution on the lunar surface. Besides the CCRs, among the limiting factors to performance are atmospheric turbulence and model uncertainties. With new CCRs on the Moon, the new differential LLR range data with 30 μm-level accuracies will enable a factor of over 100 gain compared to the present state. The new LLR facility at the TMO together with a wider distribution of CRs on the lunar surface would significantly enhance our knowledge of the deep lunar interior, adding to the contributions from the GRAIL mission and current LLR data (Williams et al. 2014).

The new *differenced* range data accurate to 30 μm-level would allow for an expansive study of the deep lunar interior including the core-mantle boundary shape, core rotation, and boundary-layer turbulence, inner solid core detection, deep mantle tidal rigidity and dissipation in a region of suspected partial melting, asymmetrical tidal response (Zhong et al. 2012), rheology-caused frequency-dependent tidal Love numbers (Efroimsky 2012; Nimmo et al. 2012; Williams and Boggs, 2015a), and stimulation of free librations (Rambaux & Williams 2010). Differenced LLR measurements would build on and enhance the results from the GRAIL mission (e.g., providing for the deep interior: study of the fluid core, detection of inner solid core, etc.)

The existing arrays currently on the moon are distributed from the equator to the northern hemisphere of the moon (see Figure 1) and are placed with modest mutual separations relative to the lunar diameter. Such a distribution is not optimal; it limits the sensitivity of the ongoing LLR science investigations of the lunar interior. A robotic deployment of several CCR instruments in various locations on the moon will result in a wider geographic separation of these instruments on the moon, which could be done in the course of upcoming missions within the Artemis program.

A wider spread of CCRs could improve the sensitivity to rotation/orientation angles and the dependent lunar science parameters by factors of up to 2.6 for longitude and up to 4 for pole orientation. The present configuration of array locations is poor for measuring lunar tidal displacements. Tidal measurements would be very much improved by a CCR array near the center of the disk, longitude 0, latitude 0, plus arrays further from the center than the Apollo sites.

Range accuracy, data span, and distributions of earth stations and retroreflectors are important considerations for future LLR data analysis. Improved range accuracy helps all solution parameters, notably lunar geophysics, and tests of gravitation. Data span is more important for some parameters, e.g., change in the gravitational constant, *G*, precession, and station motion, than others.

Science	**Current (1 cm)**	**1 mm**	**0.1 mm**
Weak EP	$\|\Delta a/a\|<2.4\times10^{-14}$	$<10^{-14}$	10^{-15}
Strong EP	$\|\eta\|<3.4\times10^{-4}$	3×10^{-5}	3×10^{-6}
PPN parameter β	$\|\beta-1\|<7.2\times10^{-5}$	$<10^{-5}$	10^{-6}
Time variation of G	9.5×10^{-15} yr^{-1}	5×10^{-15}	$<1\times10^{-15}$
Inverse Square Law	$\|\alpha\|<3\times10^{-12}$	10^{-12}	10^{-13}

Figure 5. LLR enabled progress in test of gravity.

Increased sensitivity would allow a search for new effects due to the lunar fluid core free precession, inner core influences, and stimulation of the free rotation modes. Future possibilities include detection of an inner solid core interior to the fluid core. Major advances in gravitational physics are also expected (Turyshev 2018). Advancing LLR operations to mm-level range sensitivity would allow for vastly improved accuracy of gravitational physics parameters (i.e., tests of the Equivalence Principle (EP), variability in the gravitational constant *G*, determination of PPN β, and geodetic precession, Williams et al., 2004b, see Figure 5). Anticipated improvements in Earth geophysics and geodesy results would include the positions and rates for the Earth stations, Earth rotation, precession rate, nutation, and tidal influences on the orbit (Hofmann & Mueller, 2018, Biskupek et al., 2021).

4. *Perceived Impact on the State of Knowledge in Gravitational Physics*

Recent progress in observational cosmology has challenged Einstein's general theory of relativity (GR) as a model for gravitation in our universe. From a theoretical standpoint, the challenge is even stronger—if gravity is to be quantized, general relativity will have to be modified. This continued inability to merge gravity with quantum mechanics, together with the challenges posed by the discovery of dark energy, indicates that the pure tensor gravity of GR needs modification or augmentation. It is believed that new physics beyond GR and the Standard Model of particles and fields is needed to resolve this issue (Turyshev et al., 2009b). The kinds of new physics that can solve the problems above typically involve new physical interactions, some of which could manifest themselves as violations of the EP, variation of fundamental constants, modification of the inverse square law of gravity at various distances, and other observable effects. Each of these manifestations offers an opportunity for experimentation and could lead to a major discovery (Turyshev et al., 2007, Turyshev 2008). Advanced LLR can face these challenges.

Is the Equivalence Principle exact? Currently, the Earth-Moon-Sun system provides the best solar system arena for testing the SEP. Recent solutions using LLR data yield an EP test of $(-1.8\pm1.9)\times10^{-13}$. In combination with laboratory experiments on the weak EP the SEP violation parameter η is found to be $\eta = (4.0 \pm 4.3)\times10^{-4}$ (Turyshev & Williams, 2007). Advanced LLR will push these tests further. With 1 mm precision ranging, that will be enabled by the new instruments, the EP-violating polarization will allow an EP test with a fractional sensitivity better than 1×10^{-14}, leading to a measurement of the SEP violating parameter η to 3×10^{-5}. The science results enabled by testing the EP in space with a sensitivity better than 10^{-14} will play an important role in further our quest to understand gravity, dark energy, to establish fundamental laws of nature.

Do the Fundamental Constants of Nature vary with space and time? Modern gravitational models that attempt to "complete" general relativity at very short distances or "extend" it on very large distances ($\sim10^{28}$ cm) typically include cosmologically evolving scalar fields that lead to variability of the fundamental constants. A variation of the cosmological scale factor with epoch could lead to temporal or spatial variation of the gravitational constant, *G*. A possible variation of *G* could be related to the expansion of the universe depending on the cosmological model considered. LLR measurements currently lead the search for very small spatial or temporal gradients in the value of *G* (Williams et al., 2004; Müller, Biskupek, 2007; Biskupek et al., 2021). Analysis of LLR data (Williams et al., 2004) strongly limits variation in *G* and constrains a local (~1 AU) scale expansion of the solar system as $\dot{a}/a = -\dot{G}/G = -(6\pm7)\times10^{-13}\ \mathrm{yr}^{-1}$, including those stemming from cosmological effects. Interestingly, the achieved accuracy in $\dot{G}/G$ implies that, if this rate is representative of our cosmic history, then *G* has changed by less than 1% over the 13.7 Gyr age of the universe. The ever-expanding LLR data set and its increasing accuracy will lead to further improvements in the search for variability of *G*. With 1-mm ranging accuracy, LLR data will permit a measurement of $\dot{G}/G$ to $\sim 1\times10^{-15}\ \mathrm{yr}^{-1}$ in about three years.

Do extra dimensions or other new physics alter the inverse square law (ISL)? Many modern theories of gravity, including string, supersymmetry, and brane-world theories, suggest that new physical interactions will appear at various ranges thereby modifying Newton's gravitational ISL. It is possible that non-compact extra dimensions could produce small deviations from the ISL on interplanetary scales (Dvali et al., 2003; Turyshev et al., 2007, 2009). By far the most stringent constraints on violation of the ISL at *large distances* to date come from very precise measurements of the lunar orbit about the Earth. Analysis of the LLR data tests the gravitational ISL to 3×10^{-11} of the gravitational field strength on scales of the 385,000 km Earth-Moon distance (Müller et al., 2007, Müller et al., 2018). Advanced LLR will reach sensitivity of 10^{-12} in there years.

5. Perceived Impact to the State of Knowledge in Lunar Science

The following lunar science questions are relevant to the recent NRC report *"The Scientific Context for Exploration of the moon"* (2006): What is the deep interior structure and properties? What are the core properties? Is there an inner core? What causes strong tidal dissipation? What roles did tidal and core dissipation play in the dynamical and thermal evolution? What stimulates free librations? (See Jolliff et al., 2006). The following items are of interest:

LLR is sensitive to elastic tides in two different ways: through physical librations and displacements. Dissipation-related phase shifts are seen at several frequencies for the first, but not yet for the second. More frequencies would help characterize the dissipation-vs-tidal frequency law. Tides may have a non-spherical moon component at about 1% of degree-2 tides. They will have degree-2 and -3 shapes. There should also be frequency dependence in the elastic tidal response. Observed free libration modes should damp out in 2×10^{4} to 1×10^{6} years without stimulation, so there is some source of activity. Turbulence in the fluid core could stimulate small ongoing changes in the free librations. The future program with differential LLR, would look for signatures of an inner core. Gravitational attraction between the inner core and mantle would modify the physical librations, which would be a focus of new research enabled by the differential LLR. 30 μm measurements of the tides would investigate complexities in the tides due to an asymmetric moon and rheology.

Lunar Moment of Inertia: Tracking data on orbiting s/c gives the degree-2 gravity harmonics J_2, C_{22} (Konopliv et al., 2014). LLR yields the moment of inertia combinations $(C\text{-}A)/B$ and $(B\text{-}A)/C$. Combining the two sets gives C/MR^2, the polar moment normalized with the mass *M* and radius *R* (Williams et al., 2014). LLR is sensitive to the moment of the solid moon, without fluid core.

Elastic Tides: Elastic tidal displacements are characterized by the lunar (2nd-degree) Love numbers h_2, l_2. Tidal distortion of the 2nd-degree gravity potential and moment of inertia depend on

k_2. Love numbers depend on the elastic properties of the interior including the deeper zones where the seismic information is weakest. LLR detects tidal displacement h_2; it also detects k_2, but determination of k_2 at 1 month is currently most accurate from GRAIL (Williams et al., 2014).

Tidal Dissipation: The tidal dissipation Q is a bulk property that depends on the radial distribution of the material Qs. LLR detects two dissipation terms and puts upper limits on two others, which demonstrates a dependence of tidal Q on frequency (Williams et al., 2015). The dissipation affects both the phase and amplitude of the tides. The tidal Qs are surprisingly low, but LLR does not distinguish the location of the low-Q material. At seismic frequencies, low-Q material, suspected of being a partial melt, was found for the mantle zone just above the core and that is thought to be the source of most of the tidal dissipation. With new CCRs, this determination will be improved.

Dissipation at a Liquid-Core/Solid-Mantle Interface: A fluid core does not share the rotation axis of the solid mantle. While the lunar equator precesses, a fluid core can only weakly mimic this motion. The resulting velocity difference at the core-mantle boundary (CMB) causes a torque and dissipates energy. Several dissipation terms are considered in the LLR analysis in order to separate core and tidal dissipation. Applying Yoder's turbulent boundary layer theory (Yoder, 1995) yields upper limits for the fluid core radius (Williams et al., 2001; Williams et al, 2009b).

Inner Core: A solid inner core might exist inside the fluid core. Gravitational interactions between an inner core and the mantle could reveal its presence. Too little is known about the inner core to predict the size of the perturbation of the physical librations.

Evolution & Heating: Both tidal and core-mantle dissipation would have significantly heated the moon when it was closer to the Earth (Williams et al., 2001). Early dynamical heating could have added to radiogenic heating helping to promote convection, a dynamo. Evolution studies are aided by a good determination of energy dissipation in the moon, both tidal Q vs frequency & CMB.

Core Oblateness: The fluid core exerts torques on the mantle from fluid motion at an oblate CMB. LLR detects oblateness effects and has a determination of the fluid moment difference $C_f – A_f$. LLR can determine the CMB flattening provided a good model for tidal dissipation is available.

Fluid Core Moment of Inertia: The fluid-core moment of inertia is potentially detectable, but the present uncertainty is too large to be useful (Williams et al., 2009b). LLR derived an upper limit for core radius ~374 km (Williams et al., 2001); seismology (Weber et al., 2011) derived 330 km.

Free Librations: There are 3 observed lunar free libration modes: two are strong and one weak. These oscillations of rotation are subject to damping so their amplitudes imply active or geologically recent stimulation (Newhall et al., 1997; Rambaux & Williams, 2008, 2011). If the mode similar to Chandler wobble is stimulated by eddies at the CMB (Yoder, 1981), then such activity might be revealed as irregularities in the path of polar wobble. There are also expected to be a fluid core mode and three inner core modes that have not yet been observed.

Site Positions: The moon-centered locations of the five well-ranged CCR arrays are known with sub-meter accuracy − the most accurately known positions on the moon (Williams et al., 1996b, 2013). These positions are available as control points for current/future cartographic networks (Wagner et al., 2017). Future missions can increase the number of CCR sites.

6. Conclusions

The new CCR instruments, with its high target precision, low mass, and suitability for robotic deployment, could support multiple lunar missions. The deployment of the new CCRs would enhance performance of the existing network of the laser CCR arrays currently on the Moon by a factor of over 50, especially if several of new CCRs will be delivered in widely separated locations. The main questions that will guide the LLR efforts in fundamental/gravitational physics are summarized as: Was Einstein right about gravity? Are there more space-time dimensions? How did the Universe begin? What is dark matter? Is a new theory of matter and light needed? LLR offers a strong potential for scientific discovery, especially in the tests of general relativity, searches for new physics beyond the standard model, and searches for the dynamical effects of dark matter. As a result, our understanding of the fundamental laws of Nature will be significantly improved.

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