Topical: Entangled sources for atom interferometers and atomic clocks in space

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Abstract. Spaceborne atomic clocks and atom interferometers promise answers to fundamental physics questions, with respect to general relativity, gravitational wave astronomy, or dark matter, and can be exploited for manifold applications in the fields of Earth observation and navigation. All these endeavors require highest sensitivities, and entanglement-enhanced atomic quantum sensors offer an operation in regimes fundamentally inaccessible to conventional sensors. We outline an agenda to establish entanglement-enhanced sensors as irreplaceable components for future metrological applications in space.

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I. INTRODUCTION: ENTANGLEMENT-ENHANCED SENSITIVITY

Atomic clocks and atom interferometers exploit the quantum nature of atoms for high-precision sensing. Atom interferometers can be used to measure acceleration, rotation and gravity. In this case, each individual atom in a freely falling ensemble is brought into a quantum mechanical superposition of two quantum states that are separated in position or momentum. Atomic clocks constitute today's standard for time keeping and are routinely used as frequency standards. In contrast to atom interferometers, which use quantum mechanical superpositions of external degrees of freedom, clocks use superpositions of two internal states, like spin states. After a free evolution time, the respective quantum states in both atom interferometers and atomic clocks are recombined, leading to an interference signal that depends on the physical quantity of interest. Longer times spent by the atoms in the superposition state directly lead to higher measurement resolution, more sensitivity for atom interferometers and narrower linewidths for atomic clocks. This time can be extended, for example, in atomic fountains, drop towers with microgravity environment, parabolic flights with airplanes, ballistic missiles, or satellites. Extremely long interrogation times and corresponding resolutions can be expected by operating atomic clocks and atom interferometers in space.

There is a broad range of applications of such spaceborne quantum sensors to fundamental physics: They enable high-precision tests of Einstein's equivalence principle [1] and can be used for precise tests of general relativity [2–6]. New generations of different gravitational wave detectors are proposed based on these quantum technologies [7–9] and can also be used in the search for dark matter [10–14]. The spatial superposition inherent to atom interferometers also allows tests of the limits of a quantum-mechanical description of our world [15]. Application-oriented motivations for high-precision atomic interferometers are satellite-based [16–21] and terrestrial [22–29] Earth observation, as well as navigation [30–35]. Needless to say, atomic clocks are an enabling technology for global navigation satellite systems that can, for example, be used for geodesy [36].

However, the achievable precision does not only depend on the interrogation time. Among many technical parameters, there is a fundamental limit to the resolution imposed by intrinsic fluctuations due to the quantum nature of atoms: the *Standard Quantum Limit* (SQL). This limit is related to the number of atoms used for sensing; increasing the number of atoms by a factor of 100 improves the resolution by a factor of 10. Since the available number of particles is in many cases technically limited, or density effects eventually prohibit a further increase, methods to surpass the SQL are of great importance. Surpassing the SQL is possible if the entire ensemble of many atoms is prepared in an overall quantum state characterized by specific quantum correlations between atoms: these non-classical correlations are called *metrologically-useful quantum entanglement*.

Entanglement is a central concept in quantum theory and the unifying element of many achievements in the second quantum revolution, such as quantum computing or communication [37]. Among the entire class of entangled states, the quantum states that enable a sensitivity enhancement beyond the SQL [38] form the smaller subclass of *useful entangled* states. Maximal useful-entanglement allows a factor-of-10 improvement in resolution with a tenfold increase in the number of atoms. An ensemble of 10^6 optimally entangled atoms can achieve a 1000-fold increase in resolution compared to its unentangled counterpart.

Such entanglement enhancements are assumed in many mission studies, such as for gravitational wave detection (20 dB improvement [39]), tests of Einstein's equivalence principle

(20-40 dB improvement [40]) and atomic clocks (20 dB improvement of absolute clock stability [41–43]).

In this topical white paper, we highlight the need to strategically develop the technology of entanglement-enhanced sensors from laboratory experiments to compact sensors, and test them in microgravity environments. This development would establish entangled atomic systems as an available resource for a wide range of high-precision measurements in space. The white paper summarizes state-of-the-art sources for entanglement-enhanced atomic clocks and atom interferometers in Sec. II, and describes potential applications for inertially sensitive atom interferometers (Sec. III) and clocks (Sec. IV). Section V gives an outlook on the development of entanglement-enhanced quantum sensors for space applications.

II. SOURCES OF ENTANGLED ATOMS

A particularly robust and promising class of metrologically-useful entangled states is that of spin-squeezed states [44, 45]. The generation of such atomic samples has been demonstrated along two main directions [46]: atom-light interaction and atomic collisions. The former has been applied to generate spin-squeezed states or W states in gas cells [47, 48] and laser-cooled samples [49–52]. The current squeezing record of 20.1 dB has been obtained by exploiting the coupling between the light field in a cavity and an atomic ensemble that is trapped in the cavity [41]. While those results were obtained in alkali atoms, recent success was achieved in generating entanglement on optical clock transitions. Squeezing of 12.9 dB was created in ^{171}Yb [53], and subsequently transferred to the optical clock transition, demonstrating a metrological gain of 4.4 dB for an actual frequency measurement [43]. In trapped ions, metrologically-useful entanglement of up to 14 ions has been demonstrated [54] as well as Rabi spectroscopy of a clock transition employing two entangled ions [55]. Recently, a metrological gain of 2.02(8) dB over a classical interferometer has been demonstrated with 26 entangled ions [56].

The collisional interaction in atomic Bose-Einstein condensates can be used as a nonlinear interaction for the generation of entangled quantum states [46]. Atomic collisions can be exploited to generate entanglement between external degrees of freedom. The creation of entangled momentum pairs has been demonstrated with Helium [57–59] and Rubidium [60]. Alternatively, entanglement can be generated via atom-atom interaction in highly-controllable trapped systems, such as in a double well trap [61, 62].

Entanglement can also be generated in internal degrees of freedom. The creation of spin-squeezed states was demonstrated via the one-axis twisting evolution [44] that exploits the differing collisional properties of two spin states [63, 64]. Alternatively, spin changing collisions are used to generate spin-squeezed states [42, 65–67] or Twin-Fock states [68, 69].

An entanglement-enhanced operation of inertially-sensitive atom interferometers requires the generation of entangled momentum states that are matched to the employed states in the interferometer [70, 71]. This is possible by generating entangled momentum states directly, and by matching the states to the interferometer or vice versa. Alternatively, entanglement can be generated in internal degrees of freedom and transferred to momentum space [72].

In summary, the generation of atomic sources for entanglement-enhanced quantum sensors has recently witnessed a fast development of squeezing methods, available atomic sources, and applications to clocks and interferometers. However, the demonstrations were mostly performed in proof-of-principle laboratory experiments, and a development of compact sensors and their test in zero-gravity environments is needed to exploit the potential of entangled atomic systems for high-precision measurements in space.

III. ENTANGLEMENT-ENHANCED ATOM INTERFEROMETERS IN SPACE

Unlike classical electrostatic accelerometers, atom interferometers provide absolute measurements for inertial sensing and applications. Therefore, they are expected to be deployed in space as classical accelerometry missions are reaching their accuracy limit. The use of entangled sources for space quantum gravimetry can help improve the accuracy and long-term drifts of such devices. On the other hand, the modest sensitivity of SQL-limited interferometers is constrained on ground by several technical limitations such as the vibrational noise, putting an upper bound of few ms on the interrogation time. In space, one expects to operate atom interferometers with interrogation times longer than ten seconds, and to push their instability well below the SQL. Applications in gyroscopy, gravity gradiometry, space gravimetry are expected to benefit from such a boost.

Einstein's theory of general relativity is a cornerstone of our current understanding of the physical world at macroscopic scales. However, until today no consistent theory reconciling it with quantum field theory has been found. Most unification theories are expecting a violation of general relativity. Tests of the Einstein Equivalence Principle are thus found at the heart of numerous space missions, e.g., he Space–Time Explorer and QUantum Equivalence Space Test (STE-QUEST) [1] or the Atomic Clock Ensemble in Space (ACES) [73, 74]. As these fundamental tests target the SQL, the use of entangled atomic sources will push the state of the art.

An infrasound, spaceborne gravitational wave detector based on atom interferometers is expected to cover the frequency band between 0.1 Hz and 10 Hz, hence filling the sensitivity gap between the space-based Laser Interferometer Space Antenna (LISA) [75] and the planned third generation ground-based laser interferometer (Einstein Telescope) [76]. This will add a new capability to gravitational wave astronomy that could answer long-standing questions on cosmology involving dark energy, the equivalence principle, cosmic inflation, and a grand unified theory. To access the required strain sensitivities relevant for these observatories, it is necessary to have atomic sensors operating at the $1\,\mu\mathrm{rad}/\sqrt{\mathrm{Hz}}$. This can be translated to a flux of ultra-cold atoms of $10^{12}\,\mathrm{s}^{-1}$ [39], which exceeds the state of the art by a few orders of magnitude and motivates using entangled sources to surpass the SQL.

Matter-wave experiments have so far confirmed the validity of quantum mechanics, but they do not yet rule out hypothetical modifications of the theory that would prohibit quantum superpositions, alleviate the measurement problem, and reinstate classical realism at macroscopic size and mass scales. Spontaneous collapse models form a well-studied and minimally invasive class of such modifications [77], predicting the loss of spatial coherence in many-body systems at a rate that amplifies with mass, possibly as a consequence of gravity [78–83]. Currently, the strongest parameter bounds on collapse models are achieved in optomechanical noise sensors such as LISA Pathfinder [84]. The superposition of a large mass over spatially separated modes, interfering heavy compound particles or maximally entangled NOON states of atom condensates, would be a highly sensitive experimental test [85]. The collapse rate would grow with the square of the total system mass. A similar mass dependence could be reached in a more practical scenario with squeezed, interacting quantum gases in spatially overlapping modes, provided that the interaction strength can be controlled and individual atoms can be detected with high precision [15]. A space-based experiment could offer long interrogation times, low interaction strengths, and a low level of environmental decoherence, thus probing collapse models to an unprecedented degree.

IV. ENTANGLEMENT-ENHANCED ATOMIC CLOCKS IN SPACE

Atomic clocks are at the core of current time-keeping systems and have been implementing the International System of Units' definition of a second for over 50 years [86]. Since then, various approaches towards high-precision measurements have been developed, such as microwave atomic clocks, atomic fountain clocks [87], ion clocks and optical lattice clocks [88]. Besides their relevance for novel technologies, they have evolved into a key player for ground-based tests of fundamental physics, such as general relativity [89], gravity [90], Lorentz symmetry [91] or the search for variations of the fine structure constant [92, 93] and dark matter [94]. From early on, atomic clocks have been used in airborne experiments to observe effects of the theory of relativity [95, 96]. Spaceborne missions that test general relativity are one of the milestones of modern physics and technology [3, 4]. Moreover, they are an enabling technology for global navigation satellite systems with direct implications on geodesy [36]. In such satellite systems, they constitute an already implemented network [97] with a wide range of applications, also in fundamental physics [8, 12, 14]. In fact, the most accurate test of the universality of gravitational redshift, one of the key assumptions of general relativity, has been tested with clocks upon highly eccentric satellites from global navigation satellite systems [5, 6]. These inspiring developments have to be seen in light of a strong drive towards ambitious space missions [98] to demonstrate novel technologies of atomic clocks in orbit or space and test fundamental physics, such as the Atomic Clock Ensemble in Space (ACES) [73, 74], the Cold Atom Clock Experiment in Space (CACES) [99], the Deep Space Atomic Clock (DSAC) [100] or the development of the Space Optical Clock on ISS (SOC-ISS) [101]. Spaceborne clocks have already played a crucial role for proposals in the past [1, 102] and are key to current ones such as the Space Atomic Gravity Explorer (SAGE) [103] or Fundamental physics with an Optical Clock Orbiting in Space (FOCUS).

Atom interferometers discussed in Sec. III and atomic clocks are both quantum sensors that share the same fundamental working principle. Both concepts can be combined [104–106] to interfere internal and motional states simultaneously, giving rise to new types of tests of time dilation with quantum systems [107–109]. Extensions of such schemes to differential measurements [110] are targeted at an operation close to the SQL.

As explained in Sec. I, the precision of atomic clocks using independent atoms is bounded by the SQL. Consequently, the enhancement of sensitivity of atomic sensors [46, 111] also encompasses entanglement-assisted atomic clocks as one of the prime applications. Entanglement-enhanced atomic clocks have been demonstrated in proof-of-principle experiments [41, 42, 112–115] in various systems and using different entanglement-generation methods explained in Sec. II. These experiments have clearly demonstrated the possibility to overcome the SQL of phase sensitivity for a relatively short clock-interrogation time.

To take full advantage of entanglement-based schemes, it is essential to operate atomic clocks close to the SQL. In many applications, the current limit to sensitivity is noise, for example, caused by the interrogation laser. However, many ambitious experiments of fundamental physics require differential measurements, where common-mode effects are suppressed. Nevertheless, to compete with the best conventional state-of-the-art clocks, it is necessary i) to develop entanglement generation methods that are fast enough to not substantially add dead times [116]; ii) to protect entangled states against decoherence sources [117] such that the use of entanglement does not pose limitations to the interrogation time; and iii) to develop readout protocols that optimally exploit the phase information encoded in entangled states [118–124]. Entanglement-assisted strategies are still at the focus of state-of-the-art research for a new generation of atomic clocks [43, 125] and have not yet evolved into a technique that is employed in metrological sensors.

Especially ambitions space missions, along the lines of the ongoing and planned efforts sketched above, require minimizing technical noise. Some of the proposed techniques also rely on differential measurements with common-mode suppression of deleterious effects. Every experiment will therefore try to operate close to the SQL or increase the sensitivity by other means. Consequently, spaceborne setups that are targeted at fundamental research will benefit from the implementation of sources of entangled atoms. As such, we expect entanglement-enhanced atomic clocks to play a crucial role in future missions targeted at gravitational wave detection [8, 9], dark matter searches [12], tests of general relativity [5, 6], or other fundamental physics and applications.

V. CONCLUSION

In conclusion, the broad range of applications of atom interferometers and atomic clocks, ranging from fundamental tests of physics to Earth observation and navigation, requires improvements of their sensitivity. Entangled atoms offer such an improvement, as they enable measurements with a resolution beyond the Standard Quantum Limit. The entanglement enhancement can be exploited in various directions: it is possible to obtain a higher resolution while maintaining the number of employed atoms. Furthermore, it is possible to obtain the same resolution at reduced atom number, possibly mitigating systematic density effects. Alternatively, the same resolution can be acquired in shorter averaging time, resulting in an increased measurement bandwidth. Finally, entanglement enhancement can be employed to reduce the interrogation time, which may lead to more compact set-ups, important for space applications.

Apart from the achievable improvements in the resolution, entangled atomic systems also provide qualitatively different probes for fundamental tests of physics. Entangled multi-particle states are specifically sensitive to decoherence, and this sensitivity can be exploited to search for fundamental sources of decoherence, for example in the framework of spontaneous collapse theories. Furthermore, entangled atomic states may serve as outstanding probes of the gravitational interaction. Spatially separated entangled atomic ensembles may sense the effect of general relativity on quantum-mechanical objects on scales, where the effects of spacetime curvature are not negligible. Thereby, such systems offer one of the few approaches for experimental tests of how quantum-mechanical objects couple to post-Newtonian gravity.

Leveraging the potential of entanglement-enhanced quantum sensors for space applications poses a number of challenges for the near future. i) Entangled atomic sources, so far mostly demonstrated in proof-of-principle experiments, must be integrated with high-precision interferometers and atomic clocks to exploit the demonstrated 20 dB squeezing on ground. ii) The technology must be integrated into more compact, robust sensors with low power consumption. iii) Entanglement-enhanced sensitivity must be demonstrated in a zero-gravity environment, following the pioneering experiments on drop towers, zero-g elevators, parabolic flights, ballistic missiles, satellites, and the International Space Station. An exciting perspective would be a future extension of the BECCAL program to demonstrate the readiness of the technology on the apparatus at the International Space Station. iv) An early consideration of entangled sources in the design phase of satellite missions with atomic sensors. The presented program will establish entanglement-enhanced metrology as an invaluable resource for future high-precision measurements with atomic quantum sensors in space.

[1] B. Altschul, Q. G. Bailey, L. Blanchet, K. Bongs, P. Bouyer, L. Cacciapuoti, S. Capozziello, N. Gaaloul, D. Giulini, J. Hartwig, L. Iess, P. Jetzer, A. Landragin, E. M. Rasel, S. Reynaud, S. Schiller, C. Schubert, F. Sorrentino, U. Sterr, J. D. Tasson, G. M. Tino, P. Tuckey, and P. Wolf, "Quantum tests of the einstein equivalence principle with the STE–QUEST space mission," Adv. Space Res. **55**, 501 (2015).

[2] S. Dimopoulos, P. W. Graham, J. M. Hogan, and M. A. Kasevich, "General relativistic effects in atom interferometry," Phys. Rev. D **78**, 042003 (2008).

[3] R. F. C. Vessot and M. W. Levine, "A test of the equivalence principle using a space-borne clock," Gen. Relativ. Gravitation **10**, 181 (1979).

[4] R. F. C. Vessot, M. W. Levine, E. M. Mattison, E. L. Blomberg, T. E. Hoffman, G. U. Nystrom, B. F. Farrel, R. Decher, P. B. Eby, C. R. Baugher, J. W. Watts, D. L. Teuber, and F. D. Wills, "Test of relativistic gravitation with a space-borne hydrogen maser," Phys. Rev. Lett. **45**, 2081 (1980).

[5] P. Delva, N. Puchades, E. Schönemann, F. Dilssner, C. Courde, S. Bertone, F. Gonzalez, A. Hees, Ch. Le Poncin-Lafitte, F. Meynadier, R. Prieto-Cerdeira, B. Sohet, J. Ventura-Traveset, and P. Wolf, "Gravitational redshift test using eccentric Galileo satellites," Phys. Rev. Lett. **121**, 231101 (2018).

[6] P. Delva, N. Puchades, E. Schönemann, F. Dilssner, C. Courde, S. Bertone, F. Gonzalez, A. Hees, C. Le Poncin-Lafitte, F. Meynadier, R. Prieto-Cerdeira, B. Sohet, J. Ventura-Traveset, and P. Wolf, "A new test of gravitational redshift using Galileo satellites: The GREAT experiment," C. R. Acad. Sci. **20**, 176 – 182 (2019).

[7] S. Dimopoulos, P. W. Graham, J. M. Hogan, M. A. Kasevich, and S. Rajendran, "Gravitational wave detection with atom interferometry," Phys. Lett. B **678**, 37–40 (2009).

[8] S. Kolkowitz, I. Pikovski, N. Langellier, M. D. Lukin, R. L. Walsworth, and J. Ye, "Gravitational wave detection with optical lattice atomic clocks," Phys. Rev. D **94**, 124043 (2016).

[9] M. A. Norcia, J. R. K. Cline, and J. K. Thompson, "Role of atoms in atomic gravitational-wave detectors," Phys. Rev. A **96**, 042118 (2017).

[10] A. A. Geraci and A. Derevianko, "Sensitivity of atom interferometry to ultralight scalar field dark matter," Phys. Rev. Lett. **117**, 261301 (2016).

[11] A. Derevianko and M. Pospelov, "Hunting for topological dark matter with atomic clocks," Nature Phys. **10**, 933 (2014).

[12] B. M. Roberts, G. Blewitt, C. Dailey, M. Murphy, M. Pospelov, A. Rollings, J. Sherman, W. Williams, and A. Derevianko, "Search for domain wall dark matter with atomic clocks on board global positioning system satellites," Nat. Commun. **8**, 1195 (2017).

[13] A. Arvanitaki, P. W. Graham, J. M. Hogan, S. Rajendran, and K. Van Tilburg, "Search for light scalar dark matter with atomic gravitational wave detectors," Phys. Rev. D **97**, 075020 (2018).

[14] B. M. Roberts, P. Delva, A. Al-Masoudi, A. Amy-Klein, C. Bærentsen, C. F. A. Baynham, E. Benkler, S. Bilicki, S. Bize, W. Bowden, J. Calvert, V. Cambier, E. Cantin, E. A. Curtis, S Dörscher, M. Favier, F. Frank, P. Gill, R. M. Godun, G. Grosche, C. Guo, A. Hees, I. R. Hill, R. Hobson, N. Huntemann, J. Kronjäger, S. Koke, A. Kuhl, R. Lange, T. Legero, B. Lipphardt, C. Lisdat, J. Lodewyck, O. Lopez, H. S. Margolis, H. Álvarez-Martínez, F. Mey-

nadier, F. Ozimek, E. Peik, P.-E. Pottie, N. Quintin, C. Sanner, L. De Sarlo, M. Schioppo, R. Schwarz, A. Silva, U. Sterr, Chr. Tamm, R. Le Targat, P. Tuckey, G. Vallet, T. Waterholter, D. Xu, and P. Wolf, New J. Phys. **22**, 093010 (2020).

[15] B. Schrinski, K. Hornberger, and S. Nimmrichter, "How to rule out collapse models with bec interferometry," arXiv:2008.13580 (2020), 2008.13580v1.

[16] A. Trimeche, B. Battelier, D. Becker, A. Bertoldi, P. Bouyer, C. Braxmaier, E. Charron, R. Corgier, M. Cornelius, K. Douch, N. Gaaloul, S. Herrmann, J. Müller, E. Rasel, C. Schubert, H. Wu, and F. Pereira dos Santos, "Concept study and preliminary design of a cold atom interferometer for space gravity gradiometry," Class. Quantum Grav. **36**, 245004 (2019).

[17] K. Douch, H. Wu, C. Schubert, J. Müller, and F. Pereira dos Santos, "Simulation-based evaluation of a cold atom interferometry gradiometer concept for gravity field recovery," Adv. Space Res. **61**, 1307 (2018).

[18] O. Carraz, C. Siemes, L. Massotti, R. Haagmans, and P. Silvestrin, "A spaceborne gravity gradiometer concept based on cold atom interferometers for measuring earth's gravity field," Microgravity Sci. Technol. **26**, 139 (2014).

[19] S.-w. Chiow, J. Williams, and N. Yu, "Laser-ranging long-baseline differential atom interferometers for space," Phys. Rev. A **92**, 063613 (2015).

[20] T. Lévèque, C. Fallet, M. Mandea, R. Biancale, J. M. Lemoine, S. Tardivel, M. Delpech, G. Ramillien, J. Panet, S. Bourgogne, F. Pereira Dos Santos, and Ph. Bouyer, "Correlated atom accelerometers for mapping the Earth gravity field from Space," in *International Conference on Space Optics*, Vol. 11180, edited by Zoran Sodnik, Nikos Karafolas, and Bruno Cugny, International Society for Optics and Photonics (SPIE, 2019) p. 344.

[21] T. Lévèque, C. Fallet, M. Mandea, R. Biancale, J. M. Lemoine, S. Tardivel, S. Delavault, A. Piquereau, S. Bourgogne, F. Pereira Dos Santos, B. Battelier, and P. Bouyer, "Gravity field mapping using laser-coupled quantum accelerometers in space," J. Geod. **95**, 15 (2021).

[22] S. Abend, M. Gebbe, M. Gersemann, H. Ahlers, H. Müntinga, E. Giese, N. Gaaloul, C. Schubert, C. Lämmerzahl, W. Ertmer, W. P. Schleich, and E. M. Rasel, "Atom-chip fountain gravimeter," Phys. Rev. Lett. **117**, 203003 (2016).

[23] C. Freier, M. Hauth, V. Schkolnik, B. Leykauf, M. Schilling, H. Wziontek, H.-G. Scherneck, J. Müller, and A. Peters, "Mobile quantum gravity sensor with unprecedented stability," J. Phys.: Conf. Ser. **723**, 012050 (2016).

[24] K. S. Hardman, P. J. Everitt, G. D. McDonald, P. Manju, P. B. Wigley, M. A. Sooriyabandara, C. C. N. Kuhn, J. E. Debs, J. D. Close, and N. P. Robins, "Simultaneous precision gravimetry and magnetic gradiometry with a Bose-Einstein condensate: A high precision, quantum sensor," Phys. Rev. Lett. **117**, 138501 (2016).

[25] Y. Bidel, N. Zahzam, A. Bresson, C. Blanchard, M. Cadoret, A. V. Olesen, and R. Forsberg, "Absolute airborne gravimetry with a cold atom sensor," J. Geod. **94**, 1432 (2020).

[26] Y. Bidel, N. Zahzam, C. Blanchard, A. Bonnin, M. Cadoret, A. Bresson, D. Rouxel, and M. F. Lequentrec-Lalancette, "Absolute marine gravimetry with matter-wave interferometry," Nat. Commun. **9**, 2041 (2018).

[27] X. Wu, Z. Pagel, B. S. Malek, T. H. Nguyen, F. Zi, D. S. Scheirer, and H. Müller, "Gravity surveys using a mobile atom interferometer," Sci. Adv. **5**, eaax0800 (2019).

[28] V. Ménoret, P. Vermeulen, N. Le Moigne, S. Bonvalot, P. Bouyer, A. Landragin, and B. Desruelle, "Gravity measurements below $10^{-9}\,g$ with a transportable absolute quantum gravimeter," Sci. Rep. **8**, 12300 (2018).

[29] R. Karcher, A. Imanaliev, S. Merlet, and F. Pereira Dos Santos, "Improving the accuracy of atom interferometers with ultracold sources," New J. Phys. **20**, 113041 (2018).

[30] Matthias Gersemann, Martina Gebbe, Sven Abend, Christian Schubert, and Ernst M. Rasel, "Differential interferometry using a Bose-Einstein condensate," Eur. Phys. J. D **74**, 203 (2020).

[31] C. Jekeli, "Navigation error analysis of atom interferometer inertial sensor," Navigation **52**, 1 (2005).

[32] B. Canuel, F. Leduc, D. Holleville, A. Gauguet, J. Fils, A. Virdis, A. Clairon, N. Dimarcq, Ch. J. Bordé, A. Landragin, and P. Bouyer, "Six-axis inertial sensor using cold-atom interferometry," Phys. Rev. Lett. **97**, 010402 (2006).

[33] P. Cheiney, L. Fouché, S. Templier, F. Napolitano, B. Battelier, P. Bouyer, and B. Barrett, "Navigation-compatible hybrid quantum accelerometer using a Kalman filter," Phys. Rev. Applied **10**, 034030 (2018).

[34] B. Barrett, P. Cheiney, B. Battelier, F. Napolitano, and P. Bouyer, "Multidimensional atom optics and interferometry," Phys. Rev. Lett. **122**, 043604 (2019).

[35] A. V. Rakholia, H. J. McGuinness, and G. W. Biedermann, "Dual-axis high-data-rate atom interferometer via cold ensemble exchange," Phys. Rev. Applied **2**, 054012 (2014).

[36] T. E. Mehlstäubler, G. Grosche, C. Lisdat, P. O. Schmidt, and H. Denker, "Atomic clocks for geodesy," Rep. Prog. Phys. **81**, 064401 (2018).

[37] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

[38] L. Pezzé and A. Smerzi, "Entanglement, nonlinear dynamics, and the Heisenberg limit," Phys. Rev. Lett. **102**, 100401 (2009).

[39] B. Canuel, S. Abend, P. Amaro-Seoane, F. Badaracco, Q. Beaufils, A. Bertoldi, K. Bongs, P. Bouyer, C. Braxmaier, W. Chaibi, N. Christensen, F. Fitzek, G. Flouris, N. Gaaloul, S. Gaffet, C. L. Garrido Alzar, R. Geiger, S. Guellati-Khelifa, K. Hammerer, J. Harms, J. Hinderer, M. Holynski, J. Junca, S. Katsanevas, C. Klempt, C. Kozanitis, M. Krutzik, A. Landragin, I. Làzaro Roche, B. Leykauf, Y.-H. Lien, S. Loriani, S. Merlet, M. Merzougui, M. Nofrarias, P. Papadakos, F. Pereira dos Santos, A. Peters, D. Plexousakis, M. Prevedelli, E. M. Rasel, Y. Rogister, S. Rosat, A. Roura, D. O. Sabulsky, V. Schkolnik, D. Schlippert, C. Schubert, L. Sidorenkov, J.-N. Siemß, C. F. Sopuerta, F. Sorrentino, C. Struckmann, G. M. Tino, G. Tsagkatakis, A. Viceré, W. von Klitzing, L. Woerner, and X. Zou, "ELGAR—a European laboratory for gravitation and atom-interferometric research," Class. Quantum Grav. **37**, 225017 (2020).

[40] S. Dimopoulos, P. W. Graham, J. M. Hogan, and A. Kasevich, "Testing general relativity with atom interferometry," Phys. Rev. Lett. **98**, 111102 (2007).

[41] O. Hosten, N. J. Engelsen, R. Krishnakumar, and M. A. Kasevich, "Measurement noise 100 times lower than the quantum-projection limit using entangled atoms," Nature **529**, 505 (2016).

[42] I. Kruse, K. Lange, J. Peise, B. Lücke, L. Pezzè, J. Arlt, W. Ertmer, C. Lisdat, L. Santos, A. Smerzi, and C. Klempt, "Improvement of an atomic clock using squeezed vacuum," Phys. Rev. Lett. **117**, 143004 (2016).

[43] E. Pedrozo-Peñafiel, S. Colombo, C. Shu, A. F. Adiyatullin, Z. Li, E. Mendez, B. Braverman, A. Kawasaki, D. Akamatsu, Y. Xiao, and V. Vuletić, "Entanglement on an optical atomic-clock transition," Nature **588**, 414 (2020).

[44] M. Kitagawa and M. Ueda, "Squeezed spin states," Phys. Rev. A **47**, 5138 (1993).

[45] J. Ma, X. Wang, C. Sun, and F. Nori, "Quantum spin squeezing," Phys. Rep. **509**, 89 (2011).

[46] L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, "Quantum metrology with nonclassical states of atomic ensembles," Rev. Mod. Phys. **90**, 035005 (2018).

[47] J. Hald, J. L. Sørensen, C. Schori, and E. S. Polzik, "Spin squeezed atoms: A macroscopic entangled ensemble created by light," Phys. Rev. Lett. **83**, 1319 (1999).

[48] J. Appel, P. J. Windpassinger, D. Oblak, U. B. Hoff, N. Kærgaard, and E. S. Polzik, "Mesoscopic atomic entanglement for precision measurements beyond the standard quantum limit," Proc. Natl. Acad. Sci. U. S. A. **106**, 10960 (2009).

[49] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, "States of an ensemble of two-level atoms with reduced quantum uncertainty," Phys. Rev. Lett. **104**, 073604 (2010).

[50] Z. Chen, J. Bohnet, S. Sankar, J. Dai, and J. Thompson, "Conditional spin squeezing of a large ensemble via the vacuum Rabi splitting," Phys. Rev. Lett. **106**, 133601 (2011).

[51] R. J. Sewell, M. Koschorreck, M. Napolitano, B. Dubost, N. Behbood, and M. W. Mitchell, "Magnetic sensitivity beyond the projection noise limit by spin squeezing," Phys. Rev. Lett. **109**, 253605 (2012).

[52] F. Haas, J. Volz, R. Gehr, J. Reichel, and J. Estève, "Entangled states of more than 40 atoms in an optical fiber cavity," Science **344**, 180–183 (2014).

[53] B. Braverman, A. Kawasaki, E. Pedrozo-Peñafiel, S. Colombo, C. Shu, Z. Li, E. Mendez, M. Yamoah, L. Salvi, D. Akamatsu, Y. Xiao, and V. Vuletić, "Near-unitary spin squeezing in Yb171," Phys. Rev. Lett. **122**, 223203 (2019).

[54] T. Monz, P. Schindler, J. T. Barreiro, M. Chwalla, D. Nigg, W. A. Coish, M. Harlander, W. Hänsel, M. Hennrich, and R. Blatt, "14-Qubit entanglement: Creation and coherence," Phys. Rev. Lett. **106**, 130506 (2011).

[55] R. Shaniv, T. Manovitz, Y. Shapira, N. Akerman, and R. Ozeri, "Toward Heisenberg-limited Rabi spectroscopy," Phys. Rev. Lett. **120**, 243603 (2018).

[56] C. D. Marciniak, T. Feldker, I. Pogorelov, R. Kaubruegger, D. V. Vasilyev, R. van Bijnen, P. Schindler, P. Zoller, R. Blatt, and T. Monz, "Optimal metrology with variational quantum circuits on trapped ions," arXiv:2107.01860 (2021), 2107.01860.

[57] M. Bonneau, J. Ruaudel, R. Lopes, J.-C. Jaskula, A. Aspect, D. Boiron, and C. I. Westbrook, "Tunable source of correlated atom beams," Phys. Rev. A **87**, 061603 (2013).

[58] D. K. Shin, B. M. Henson, S. S. Hodgman, T. Wasak, J. Chwedeńczuk, and A. G. Truscott, "Bell correlations between spatially separated pairs of atoms," Nat. Commun. **10**, 4447 (2019).

[59] M. Keller, M. Kotyrba, F. Leupold, M. Singh, M. Ebner, and A. Zeilinger, "Bose-Einstein condensate of metastable helium for quantum correlation experiments," Phys. Rev. A **90**, 063607 (2014).

[60] R. Bucker, J. Grond, S. Manz, T. Berrada, T. Betz, C. Koller, U. Hohenester, T. Schumm, A. Perrin, and J. Schmiedmayer, "Twin-atom beams," Nature Phys. **7**, 608 (2011).

[61] J. Estève, C. Gross, A. Weller, S. Giovanazzi, and M. K. Oberthaler, "Squeezing and entanglement in a Bose-Einstein condensate," Nature **455**, 1216 (2008).

[62] T. Berrada, S. van Frank, R. Bücker, T. Schumm, J.-F. Schaff, and J Schmiedmayer, "Integrated Mach-Zehnder interferometer for Bose-Einstein condensates," Nat. Commun. **4**, 2077 (2013).

[63] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, "Nonlinear atom interfer-

ometer surpasses classical precision limit," Nature **464**, 1165 (2010).

[64] M. Riedel, P. Böhi, Y. Li, T. Hänsch, A. Sinatra, and P. Treutlein, "Atom-chip-based generation of entanglement for quantum metrology," Nature **464**, 1170 (2010).

[65] C. Gross, H. Strobel, E. Nicklas, T. Zibold, N. Bar-Gill, G. Kurizki, and M. K. Oberthaler, "Atomic homodyne detection of continuous-variable entangled twin-atom states," Nature **480**, 219 (2011).

[66] C. D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans, and M. S. Chapman, "Spin-nematic squeezed vacuum in a quantum gas," Nature Phys. **8**, 305 (2012).

[67] J. Peise, I. Kruse, K. Lange, B. Lücke, L. Pezzè, J. Arlt, W. Ertmer, K. Hammerer, L. Santos, A. Smerzi, and C. Klempt, "Satisfying the Einstein-Podolsky-Rosen criterion with massive particles," Nat. Commun. **6**, 8984 (2015).

[68] B. Lücke, M. Scherer, J. Kruse, L. Pezzé, F. Deuretzbacher, P. Hyllus, O. Topic, J. Peise, W. Ertmer, J. Arlt, L. Santos, A. Smerzi, and C. Klempt, "Twin matter waves for interferometry beyond the classical limit," Science **334**, 773 (2011).

[69] X.-Y. Luo, Y.-Q. Zou, L.-N. Wu, Q. Liu, M.-F. Han, M. K. Tey, and L. You, "Deterministic entanglement generation from driving through quantum phase transitions," Science **355**, 620 (2017).

[70] S. S. Szigeti, S. P. Nolan, J. D. Close, and S. A. Haine, "High-precision quantum-enhanced gravimetry with a bose-einstein condensate," Phys. Rev. Lett. **125**, 100402 (2020).

[71] Robin Corgier, Naceur Gaaloul, Augusto Smerzi, and Luca Pezzè, "Delta-kick squeezing," Phys. Rev. Lett. **127**, 183401 (2021).

[72] F. Anders, A. Idel, P. Feldmann, D. Bondarenko, S. Loriani, K. Lange, J. Peise, M. Gersemann, B. Meyer-Hoppe, S. Abend, N. Gaaloul, C. Schubert, D. Schlippert, L. Santos, E. Rasel, and C. Klempt, "Momentum entanglement for atom interferometry," Phys. Rev. Lett. **127**, 140402 (2021).

[73] M. Lilley, E. Savalle, M. C. Angonin, P. Delva, C. Guerlin, C. Le Poncin-Lafitte, F. Meynadier, and P. Wolf, "ACES/PHARAO: high-performance space-to-ground and ground-to-ground clock comparison for fundamental physics," GPS Solut. **25**, 34 (2021).

[74] L. Cacciapuoti, M. Armano, R. Much, O. Sy, A. Helm, M. P. Hess, J. Kehrer, S. Koller, T. Niedermaier, F. X. Esnault, D. Massonnet, D. Goujon, J. Pittet, P. Rochat, S. Liu, W. Schaefer, T. Schwall, I. Prochazka, A. Schlicht, U. Schreiber, P. Delva, C. Guerlin, P. Laurent, C. le Poncin-Lafitte, M. Lilley, E. Savalle, P. Wolf, F. Meynadier, and C. Salomon, "Testing gravity with cold-atom clocks in space: The ACES mission," Eur. Phys. J. D **74**, 164 (2020).

[75] O. Jennrich, "LISA technology and instrumentation," Class. Quantum Grav. **26**, 153001 (2009).

[76] M. Punturo, M. Abernathy, F. Acernese, B. Allen, N. Andersson, K. Arun, F. Barone, B. Barr, M. Barsuglia, M. Beker, N. Beveridge, S. Birindelli, S. Bose, L. Bosi, S. Braccini, C. Bradaschia, T. Bulik, E. Calloni, G. Cella, E. Chassande Mottin, S. Chelkowski, A. Chincarini, J. Clark, E. Coccia, C. Colacino, J. Colas, A. Cumming, L. Cunningham, E. Cuoco, S. Danilishin, K. Danzmann, G. De Luca, R. De Salvo, T. Dent, R. De Rosa, L. Di Fiore, A. Di Virgilio, M. Doets, V. Fafone, P. Falferi, R. Flaminio, J. Franc, F. Frasconi, A. Freise, P. Fulda, J. Gair, G. Gemme, A. Gennai, A. Giazotto, K. Glampedakis, M. Granata, H. Grote, G. Guidi, G. Hammond, M. Hannam, J. Harms, D. Heinert, M. Hendry, I. Heng, E. Hennes, S. Hild, J. Hough, S. Husa, S. Huttner, G. Jones, F. Khalili, K. Kokeyama, K. Kokko-

tas, B. Krishnan, M. Lorenzini, H. Lück, E. Majorana, I. Mandel, V. Mandic, I. Martin, C. Michel, Y. Minenkov, N. Morgado, S. Mosca, B. Mours, H. Müller–Ebhardt, P. Murray, R. Nawrodt, J. Nelson, R. Oshaughnessy, C. D. Ott, C. Palomba, A. Paoli, G. Parguez, A. Pasqualetti, R. Passaquieti, D. Passuello, L. Pinard, R. Poggiani, P. Popolizio, M. Prato, P. Puppo, D. Rabeling, P. Rapagnani, J. Read, T. Regimbau, H. Rehbein, S. Reid, L. Rezzolla, F. Ricci, F. Richard, A. Rocchi, S. Rowan, A. Rüdiger, B. Sassolas, B. Sathyaprakash, R. Schnabel, C. Schwarz, P. Seidel, A. Sintes, K. Somiya, F. Speirits, K. Strain, S. Strigin, P. Sutton, S. Tarabrin, A. Thüring, J. van den Brand, C. van Leewen, M. van Veggel, C. van den Broeck, A. Vecchio, J. Veitch, F. Vetrano, A. Vicere, S. Vyatchanin, B. Willke, G. Woan, P. Wolfango, and K. Yamamoto, "The Einstein Telescope: A third-generation gravitational wave observatory," Class. Quantum Grav. **27**, 194002 (2010).

[77] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, "Models of wave-function collapse, underlying theories, and experimental tests," Rev. Mod. Phys. **85**, 471 (2013).

[78] L. Diosi, "A universal master equation for the gravitational violation of quantum mechanics," Phys. Lett. A **120**, 377 (1987).

[79] R. Penrose, "On gravity's role in quantum state reduction," Gen. Relativ. Gravitation **28**, 581 (1996).

[80] D. Giulini and A. Großardt, "The Schrödinger–Newton equation as a non-relativistic limit of self-gravitating Klein–Gordon and Dirac fields," Class. Quantum Grav **29**, 215010 (2012).

[81] D. Kafri, J. M. Taylor, and G. J. Milburn, "A classical channel model for gravitational decoherence," New J. Phys. **16**, 065020 (2014).

[82] A. Tilloy and L. Diósi, "Sourcing semiclassical gravity from spontaneously localized quantum matter," Phys. Rev. D **93**, 024026 (2016).

[83] L. Asprea, A. Bassi, H. Ulbricht, and G. Gasbarri, "Gravitational decoherence and the possibility of its interferometric detection," Phys. Rev. Lett. **126**, 200403 (2021).

[84] M. Carlesso, A. Bassi, P. Falferi, and A. Vinante, "Experimental bounds on collapse models from gravitational wave detectors," Phys. Rev. D **94**, 124036 (2016).

[85] G. Gasbarri, A. Belenchia, M. Carlesso, S. Donadi, A. Bassi, R. Kaltenbaek, M. Paternostro, and H. Ulbricht, "Testing the foundation of quantum physics in space via interferometric and non-interferometric experiments with mesoscopic nanoparticles," Commun. Phys. **4**, 1 (2021).

[86] E. F. Arias, D. Matsakis, T. J. Quinn, and P. Tavella, "The 50th anniversary of the atomic second," IEEE Trans. Ultrason. Ferroelectr. Freq. Control **65**, 898–903 (2018).

[87] R. Wynands and S Weyers., "Atomic fountain clocks," Metrologia **42**, S64 (2005).

[88] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, "Optical atomic clocks," Rev. Mod. Phys. **87**, 637 (2015).

[89] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, "Optical clocks and relativity," Science **329**, 1630–1633 (2010).

[90] M. Takamoto, I. Ushijima, N. Ohmae, T. Yahagi, K. Kokado, H. Shinkai, and H. Katori, "Test of general relativity by a pair of transportable optical lattice clocks," Nat. Photonics **14**, 411 (2020).

[91] C. Sanner, N. Huntemann, R. Lange, C. Tamm, E. Peik, M. S. Safronova, and S. G. Porsev, "Optical clock comparison for lorentz symmetry testing," Nature **567**, 204 (2019).

[92] V. A. Dzuba, A. Derevianko, and V. V. Flambaum, "Ion clock and search for the variation of the fine-structure constant using optical transitions in Nd^{13+} and Sm^{15+}," Phys. Rev. A

86, 054502 (2012).

[93] M. S. Safronova, S. G. Porsev, C. Sanner, and J. Ye, "Two clock transitions in neutral Yb for the highest sensitivity to variations of the fine-structure constant," Phys. Rev. Lett. **120**, 173001 (2018).

[94] C. J. Kennedy, E. Oelker, J. M. Robinson, T. Bothwell, D. Kedar, W. R. Milner, G. E. Marti, A. Derevianko, and J. Ye, "Precision metrology meets cosmology: Improved constraints on ultralight dark matter from atom-cavity frequency comparisons," Phys. Rev. Lett. **125**, 201302 (2020).

[95] J. C. Hafele and R. E. Keating, "Around-the-world atomic clocks: Predicted relativistic time gains," Science **177**, 166–168 (1972).

[96] J. C. Hafele and R. E. Keating, "Around-the-world atomic clocks: Observed relativistic time gains," Science **177**, 168–170 (1972).

[97] Fritz Riehle, "Optical clock networks," Nat. Photonics **11**, 25 (2017).

[98] C. Lämmerzahl, G. Ahlers, N. Ashby, M. Barmatz, P. L. Biermann, H. Dittus, V. Dohm, R. Duncan, K. Gibble, J. Lipa, N. Lockerbie, N. Mulders, and C. Salomon, "Review: Experiments in fundamental physics scheduled and in development for the ISS," Gen. Relativ. Gravitation **36**, 615 (2004).

[99] L. Liu, D.-S. Lü, W.-B. Chen, T. Li, Q.-Z. Qu, B. Wang, L. Li, W. Ren, Z.-R. Dong, J.-B. Zhao, W.-B. Xia, X. Zhao, J.-W. Ji, M.-F. Ye, Y.-G. Sun, Y.-Y. Yao, D. Song, Z.-G. Liang, S.-J. Hu, D.-H. Yu, X. Hou, W. Shi, H.-G. Zang, J.-F. Xiang, X.-K. Peng, and Y.-Z. Wang, "In-orbit operation of an atomic clock based on laser-cooled 87rb atoms," Nat. Commun. **9**, 2760 (2018).

[100] E. A. Burt, J. D. Prestage, R. L. Tjoelker, D. G. Enzer, D. Kuang, D. W. Murphy, D. E. Robison, J. M. Seubert, R. T. Wang, and T. A. Ely, "Demonstration of a trapped-ion atomic clock in space," Nature **595**, 43 (2021).

[101] S. Origlia, M. S. Pramod, S. Schiller, Y. Singh, K. Bongs, R. Schwarz, A. Al-Masoudi, S. Dörscher, S. Herbers, S. Häfner, U. Sterr, and Ch. Lisdat, "Towards an optical clock for space: Compact, high-performance optical lattice clock based on bosonic atoms," Phys. Rev. A **98**, 053443 (2018).

[102] S. R. Jefferts, T. P. Heavner, L. W. Hollberg, J. Kitching, D. M. Meekhof, T. E. Parker, W. Phillips, S. Rolston, H. G. Robinson, J. H. Shirley, D. B. Sullivan, F. L. Walls, N. Ashby, W. M. Klipstein, L. Maleki, D. Seidel, R. Thompson, S. Wu, L. Young, R.F.C. Vessot, and A. DeMarchi, "PARCS: a primary atomic reference clock in space," in *Proc. Joint Meeting Europ. Freq. Time Forum – IEEE Internat. Freq. Control Symp.*, Vol. 1 (1999) p. 141.

[103] G. M. Tino, A. Bassi, G. Bianco, K. Bongs, P. Bouyer, L. Cacciapuoti, S. Capozziello, X. Chen, M. L. Chiofalo, A. Derevianko, W. Ertmer, N. Gaaloul, P. Gill, P. W. Graham, J. M. Hogan, L. Iess, M. A. Kasevich, H. Katori, C. Klempt, X. Lu, L.-S. Ma, H. Müller, N. R. Newbury, C. W. Oates, A. Peters, N. Poli, E. M. Rasel, G. Rosi, A. Roura, C. Salomon, S. Schiller, W. P. Schleich, D. Schlippert, F. Schreck, C. Schubert, F. Sorrentino, U. Sterr, J. W. Thomsen, G. Vallone, F. Vetrano, P. Villoresi, W. von Klitzing, D. Wilkowski, P. Wolf, J. Ye, N. Yu, and M. Zhan, "SAGE: A proposal for a space atomic gravity explorer," Eur. Phys. J. D **73**, 228 (2019).

[104] S. Sinha and J. Samuel, "Atom interferometry and the gravitational redshift," Class. Quant. Grav. **28**, 145018 (2011).

[105] M. Zych, F. Costa, I. Pikovski, and Č. Brukner, "Quantum interferometric visibility as a

witness of general relativistic proper time," Nat. Commun. **2**, 505 (2011).

[106] I. Pikovski, M. Zych, F. Costa, and Č. Brukner, "Time dilation in quantum systems and decoherence," New J. Phys. **19**, 025011 (2017).

[107] A. Roura, "Gravitational redshift in quantum-clock interferometry," Phys. Rev. X **10**, 021014 (2020).

[108] S. Loriani, A. Friedrich, C. Ufrecht, F. Di Pumpo, S. Kleinert, S. Abend, N. Gaaloul, C. Meiners, C. Schubert, D. Tell, É. Wodey, M. Zych, W. Ertmer, A. Roura, D. Schlippert, W. P. Schleich, E. M. Rasel, and E. Giese, "Interference of clocks: A quantum twin paradox," Sci. Adv. **5**, eaax8966 (2019).

[109] C. Ufrecht, F. Di Pumpo, A. Friedrich, A. Roura, C. Schubert, D. Schlippert, E. M. Rasel, W. P. Schleich, and E. Giese, "Atom-interferometric test of the universality of gravitational redshift and free fall," Phys. Rev. Research **2**, 043240 (2020).

[110] A. Roura, C. Schubert, D. Schlippert, and E. M. Rasel, "Measuring gravitational time dilation with delocalized quantum superpositions," Phys. Rev. D **104**, 084001 (2021).

[111] S. S. Szigeti, O. Hosten, and S. A. Haine, "Improving cold-atom sensors with quantum entanglement: Prospects and challenges," Appl. Phys. Lett. **118**, 140501 (2021).

[112] V. Meyer, M. A. Rowe, D. Kielpinski, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, "Experimental demonstration of entanglement-enhanced rotation angle estimation using trapped ions," Phys. Rev. Lett. **86**, 5870 (2001).

[113] A. Louchet-Chauvet, J. Appel, J. J. Renema, D. Oblak, N. Kjaergaard, and E. S. Polzik, "Entanglement-assisted atomic clock beyond the projection noise limit," New J. Phys. **12**, 065032 (2010).

[114] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, "Orientation-dependent entanglement lifetime in a squeezed atomic clock," Phys. Rev. Lett. **104**, 250801 (2010).

[115] C. F. Ockeloen, R. Schmied, M. F. Riedel, and P. Treutlein, "Quantum metrology with a scanning probe atom interferometer," Phys. Rev. Lett. **111**, 143001 (2013).

[116] M. Schulte, C. Lisdat, P. O. Schmidt, U. Sterr, and K. Hammerer, "Prospects and challenges for squeezing-enhanced optical atomic clocks," Nat. Commun. **11**, 5955 (2020).

[117] B. M. Escher, R. L. de Matos Filho, and L. Davidovich, "General framework for estimating the ultimate precision limit in noisy quantum-enhanced metrology," Nature Phys. **7**, 406 (2011).

[118] A. André, A. S. Sørensen, and M. D. Lukin, "Stability of atomic clocks based on entangled atoms," Phys. Rev. Lett. **92**, 230801 (2004).

[119] N. Shiga and M. Takeuchi, "Locking the local oscillator phase to the atomic phase via weak measurement," New J. Phys. **14**, 023034 (2012).

[120] J. Borregaard and A. S. Sørensen, "Near-Heisenberg-limited atomic clocks in the presence of decoherence," Phys. Rev. Lett. **111**, 090801 (2013).

[121] M. Mullan and E. Knill, "Optimizing passive quantum clocks," Phys. Rev. A **90**, 042310 (2014).

[122] E. M. Kessler, P. Kómár, M. Bishof, L. Jiang, A. S. Sørensen, J. Ye, and M. D. Lukin, "Heisenberg-limited atom clocks based on entangled qubits," Phys. Rev. Lett. **112**, 190403 (2014).

[123] L. Pezzè and A. Smerzi, "Heisenberg-limited noisy atomic clock using a hybrid coherent and squeezed state protocol," Phys. Rev. Lett. **125**, 210503 (2020).

[124] L. Pezzè and A. Smerzi, "Quantum phase estimation algorithm with gaussian spin states,"

Phys. Rev. X Quantum **2**, 040301 (2021).

[125] B. K. Malia, J. Martínez-Rincón, Y. Wu, O. Hosten, and M. A. Kasevich, "Free space Ramsey spectroscopy in rubidium with noise below the quantum projection limit," Phys. Rev. Lett. **125**, 043202 (2020).