Space-borne Clocks for Geodetic Applications

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Clock networks, i.e. high-performance clocks connected with dedicated frequency links, are a novel promising tool in geodesy. They will enable "relativistic geodesy with clocks" with the required accuracy in practice: Gravity potential differences, resp. physical height differences, can be determined from clock measurements by observing the gravitational redshift effect through the ultra-precise comparison of their frequencies.

Today, the latest generation of optical clocks approaches a fractional frequency uncertainty of 1.0×10^{-18} . It corresponds to about 1.0 cm in height or $0.1 \, \text{m}^2/\text{s}^2$ in geopotential. In recent years, a number of experiments have been carried out and demonstrated that clocks can resolve the height (potential) difference between long-distance sites, i.e., from hundreds of meters to thousands of kilometers, at an accuracy level from cm to dm.

The high precision of resolving height and potential differences makes clock networks highly relevant to a variety of geodetic applications [1]. To unify local height systems, a few clocks' measurements can provide the discrepancies between local height datums and adjust the systematic distortions of national reference networks e.g., slopes along the levelling lines [2]. A hybrid clock network which consists of stationary and transportable clocks on ground and master clocks in space is conceptualized to realize a consistent, accurate and stable global International Height Reference System (IHRS) [3].

In addition, a terrestrial clock network can provide time series of point-wise gravity potential values reflecting mass variations on Earth. These novel data are complementary to the results of satellite gravity missions like GRACE/GRACE-FO. For example in Greenland, clock measurements can determine the secular mass change caused by ice melting. They are also capable to detect the seasonal hydrological variations in Amazon. In specific locations, it might even be possible to detect

time-variable gravity signals that are too small to be observed with the sensitivity of GRACE(-FO) [5].

Space-Borne Clocks

High-performance clocks in space may offer major measurement advantages for future satellite missions by providing a highly stable and accurate on-board frequency reference. For example, the Atomic Clock Ensemble in Space (ACES) mission will operate a new generation of atomic clocks Space Station International (ISS) and the performance demonstrate microwave link technique [6]. Clocks in space will facilitate applications in many fields, e.g., for testing fundamental laws of physics, in time frequency metrology, for positioning and navigation, in geodesy and gravimetry, and so on.

In the following, we will illustrate the potential application of space-based clocks for the determination of the Earth's gravity field [4]. Three mission scenarios which address different configurations of clock networks for observing gravitational potential differences over the whole Earth have been investigated: i) a ground-to-space clock comparison; iii) a space-to-space clock comparison; iii) the combined case of scenarios i) and ii).

For the first scenario, a target clock is assumed on-board of a Low Earth Orbiter (LEO) satellite to collect gravitational potential values along the orbit. In order to obtain a global data set with a homogeneous coverage that is required for the recovery of the Earth's global gravity field, the target clock has continuously to track ground-based reference clocks. The gravitational potential values along orbit are obtained through the ground-to-space clock frequency comparisons. However, it still poses a great challenge in practice to realize such worldwide distributed ground reference clocks and the free-space links.

For the scenario of "space-to-space clock comparison", we follow the basic configuration of the GRACE/GRACE-FO missions, where two identical clocks are considered on twin coorbiting satellites that are separated by a few tens or hundreds of kilometers. The two clocks can be compared to each other via free-space laser links to determine the gravitational potential difference between them. To support the real-time link between both clocks, additional relay satellites with master clocks or transponders in high orbits might be required. This scenario is illustrated in Figure 1. One major advantage of this scenario is getting rid of the dependence on reference clocks on ground.

As a combined case of the above two scenarios, some reference clocks on ground and two clocks on co-orbiting satellites are considered. This case can deliver an integrated data set of gravity potential values and gravity potential differences along the orbit. It can help to keep the individual strength of both types of observations in the final solution.

The benefit of those clock measurements for determining the Earth's gravity field is quantitatively evaluated through closed-loop simulations. The measurements are simulated along a one-month orbit with an average altitude of about 450 km, and affected by a nominal noise level of 1.0 x 10⁻¹⁸, including frequency uncertainties from both, the clocks

and the links. Moreover, one optimistic case with a noise level of 1.0×10^{-19} and one pessimistic case with a noise level of 1.0×10^{-17} are computed for a comparison. The results are shown in Figure 2. It indicates clearly that clock measurements contribute mostly to recover the very low-degree gravity field coefficients, which are not optimally estimated from previous gravity or gradiometry satellite missions. Space-borne clocks could make relevant contributions to reveal the large-scale Earth mass changes over time.

Conclusions: Clocks in space provide the unique chance to directly observe the gravitational potential (difference) rather than some derived quantities that were obtained in previous satellite gravity missions. This kind of scalar quantities can improve the determination of the very low-degree gravity field coefficients and thus reveal the large-scale mass variations on Earth.

Collaborations: Funded by the German research foundation DFG through major research programs, like CRC 1464 TerraQ or EXC 2123 QuantumFrontiers, we in Hannover have close collaboration with ZARM (Center of Applied Space Technology and Microgravity, Bremen) and PTB (National Metrology Institute of Germany, Braunschweig) to promote the development and use of clock networks for geodetic applications.

References: [1] J. Müller, D. Dirkx, SM Kopeikin, G. Lion, I. Panet, G. Petit, P. Visser, 2018, *Space Sci Rev*, doi: 10.1007/s11214-017-0431-z [2] H. Wu, J. Müller, C. Lämmerzahl, 2018, *Geophys. J. Int.*, doi: 10.1093/gji/ggy508 [3] J. Müller and H. Wu, 2020, *J Geod*, doi: 10.1007/s00190-020-01401-8 [4] H. Wu and J. Müller, 2020, *Int. Asso. of Geod. Symp.*, doi: 10.1007/1345_2020_97 [5] S. Schröder, S. Stellmer, J. Kusche, 2021, *Geophys. J. Int.*, doi:10.1093/gji/ggab132 [6] L. Cacciapuoti and C. Salomon, 2011, *J. Phys.: Conf. Ser.*, doi: 10.1088/1742-6596/327/1/012049

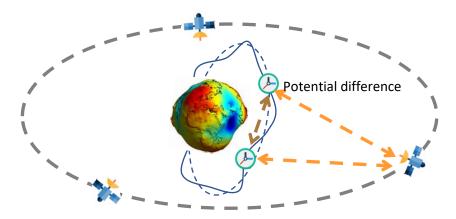


Figure 1: Space-to-space clock comparison to observe gravitational potential differences between co-orbiting satellites for the determination of the Earth's gravity field

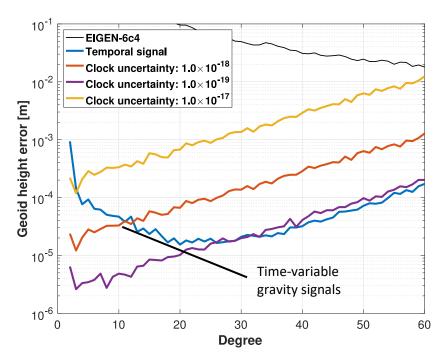


Figure 2: Comparison of monthly clock solutions that are recovered from simulated measurements assuming different magnitudes of noise, and the time-variable gravity signals to be determined