National Aeronautics and Space Administration

GDC Orbit Primer

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code 595

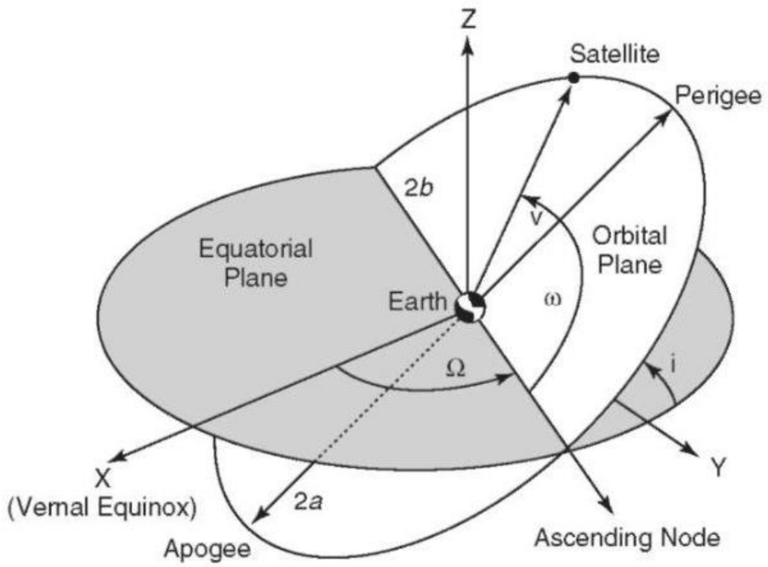
NAVIGATION & MISSION DESIGN BRANCH
NASA GSFC

Outline

- Orbital Elements
- Orbital Precession
  - Differential Rates
- String of Pearls
  - In-Plane Drift
- ΔV Calculations
- Orbital Decay
- Radiation Dosage

Earth Orbital Elements

- Six elements are needed to specify an orbit state
- Semi-major Axis (SMA, a)
  - Half the long-axis of the ellipse
  - Perigee Radius (Rp) – closest approach to Earth
  - Apogee Radius (Ra) – furthest distance from Earth
  - SMA = (Rp + Ra)/2
- Eccentricity (e) – ellipticity of the orbit
  - e = 0 ☞ Circle
  - 0 < e < 1 ☞ Ellipse
  - e = 1 ☞ Parabola
  - e = Ra/a -1 or e = 1 – Rp/a
- Inclination (i)
  - Angle orbit makes to Earth equator
  - Angle between orbit normal vector and North Pole
- Right Ascension of Ascending Node (Ω)
  - Angle in equatorial plane between Ascending Node and Vernal Equinox



- Argument of Periapsis (ω) – angle in orbit plane from ascending node to perigee
- True Anomaly (TA) – angle in orbit plane from perigee to orbital location

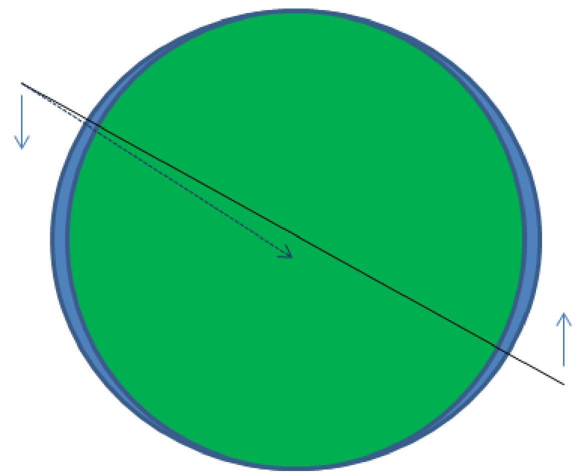
Nodal Precession (Regression)

- The Earth’s equatorial bulge imparts an out-of-plane force on the orbit which causes a gyroscopic precession of the line of nodes

$$\frac{d\Omega}{dt} \quad \frac{-3nJ_2R_E^2 cosi}{2a^2(1-e^2)^2} \text{ rad/sec}$$

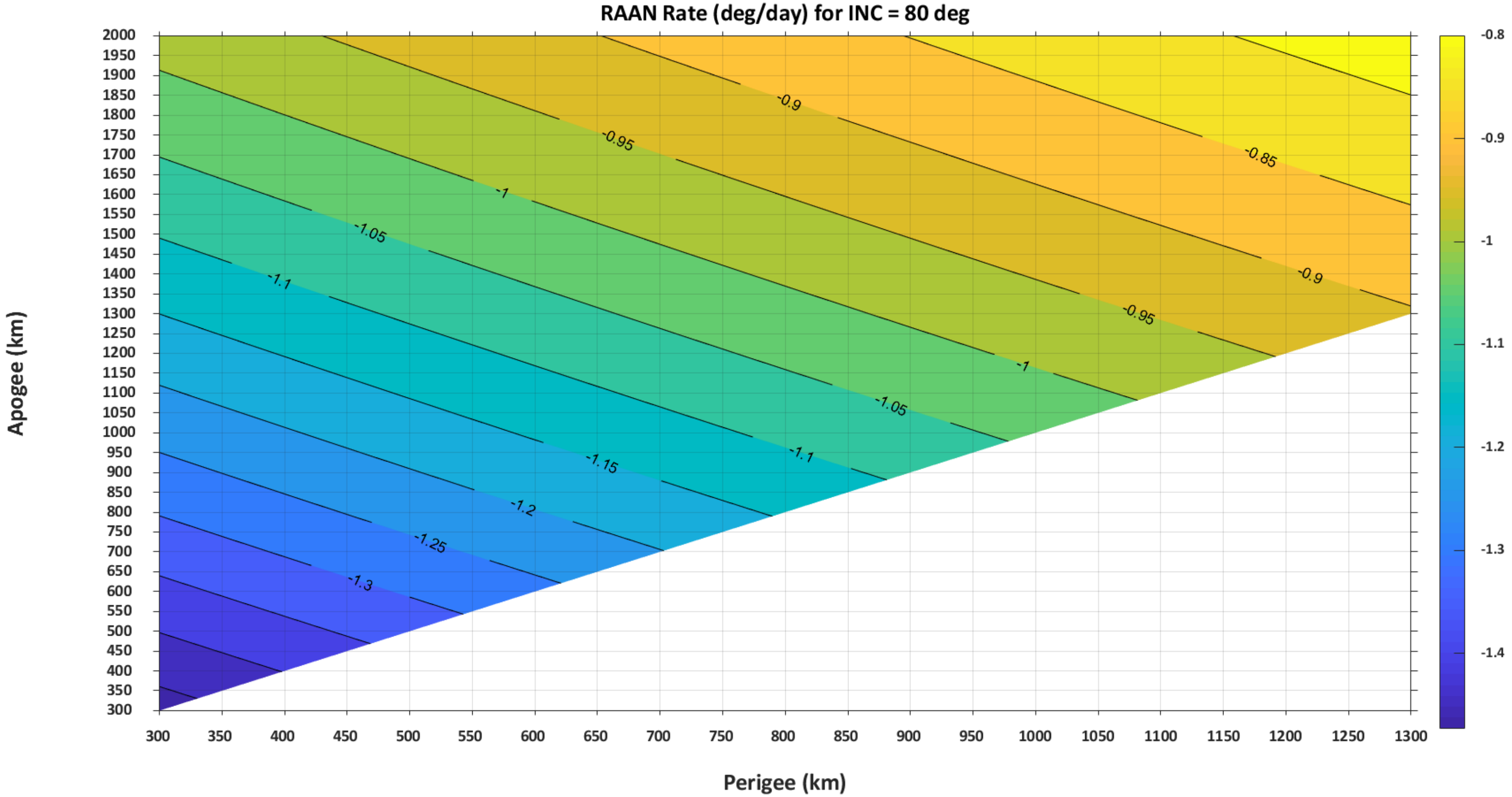
where

- R_E – Earth Radius
- n – mean motion ($\sqrt{\mu/a^3}$)
- J_2 – Geopotential term related to Earth’s oblateness

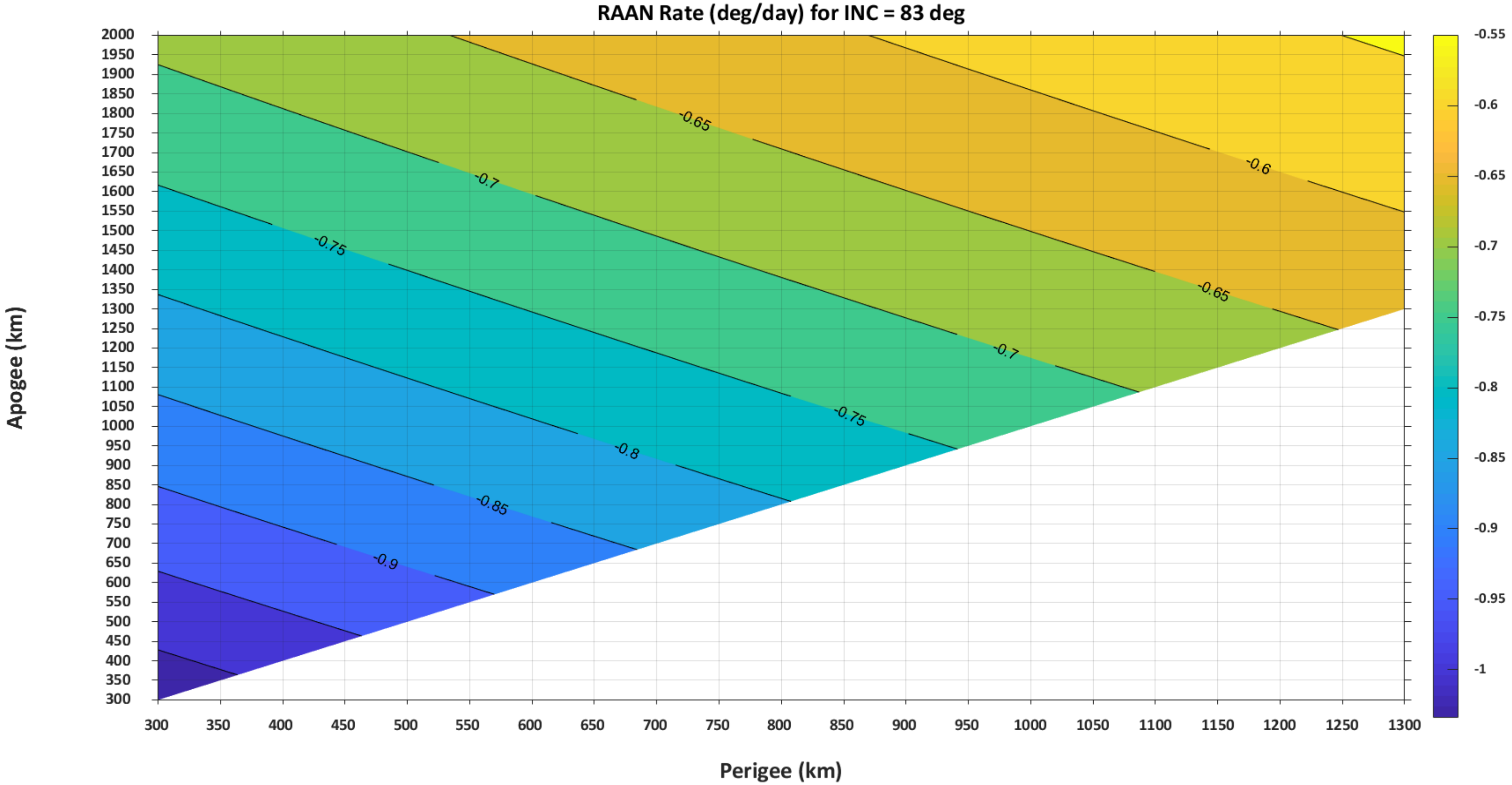


- Polar orbits have no nodal precession
- When nodal precession is equal to the apparent motion of the Sun (0.9856 deg/day … 360 deg in 365.2422 days) then we have a Sun-Synchronous orbit

Nodal Precession - 80° Inclination

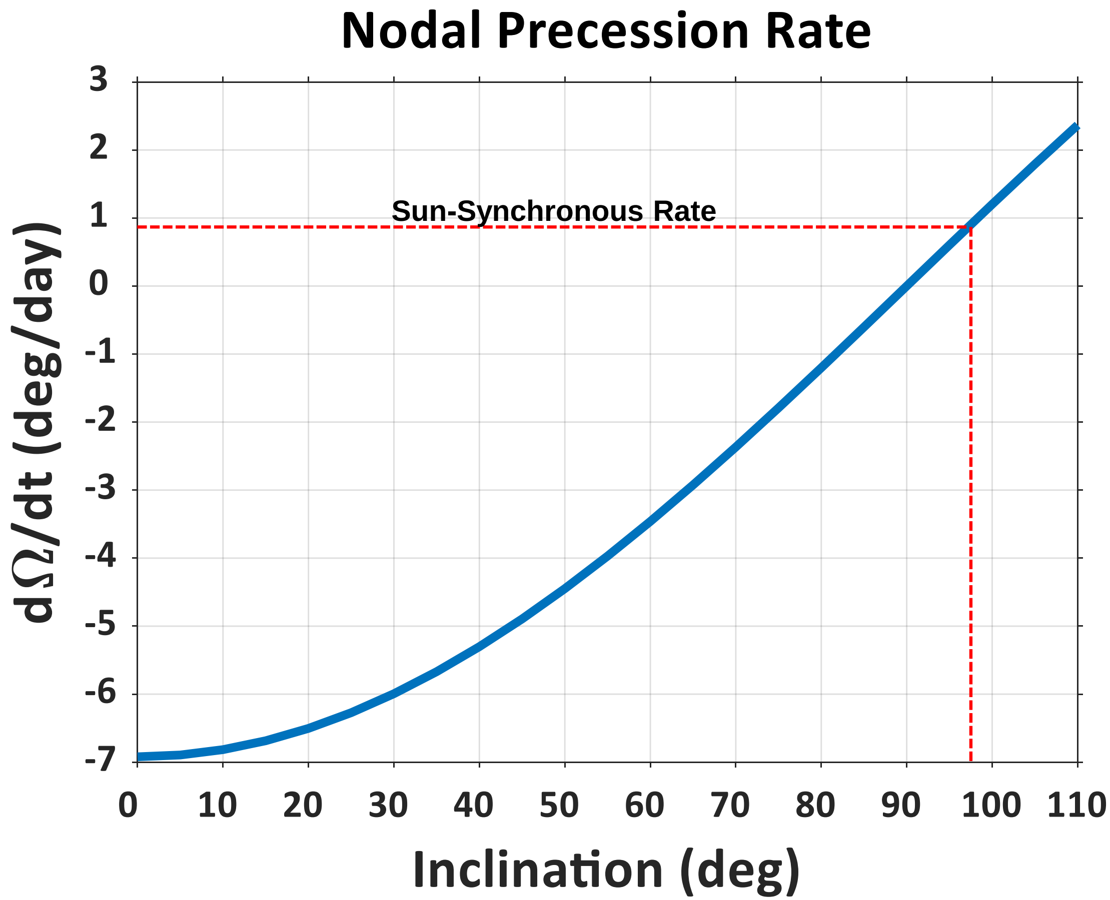


Nodal Precession - 83° Inclination



Nodal Precession – 700 km, Circular

- For a 700 km, circular orbit, the Sun-synchronous inclination is 98.2 deg



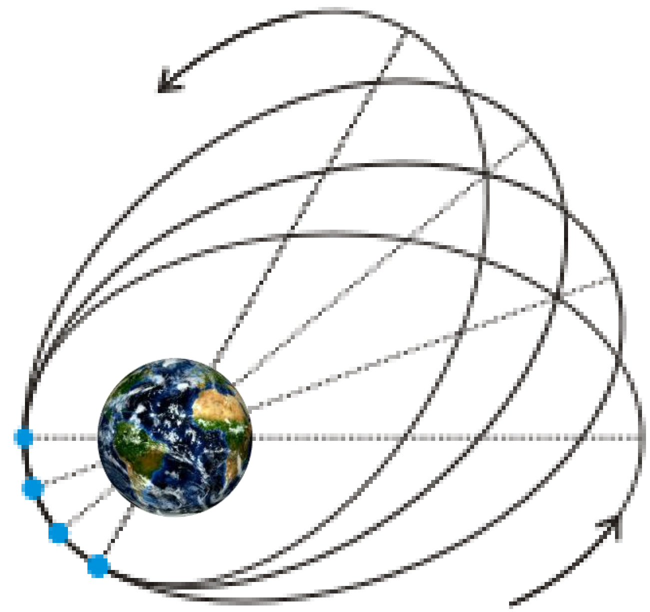
Apsidal Precession

- Similar to nodal regression, the Earth's oblateness causes a precession of the orbital line of apsides

$$\frac{d\omega}{dt} = \frac{3nJ_2R_E^2(4-5sin^2 i)}{4a^2(1-e^2)^2} \text{ rad/sec}$$

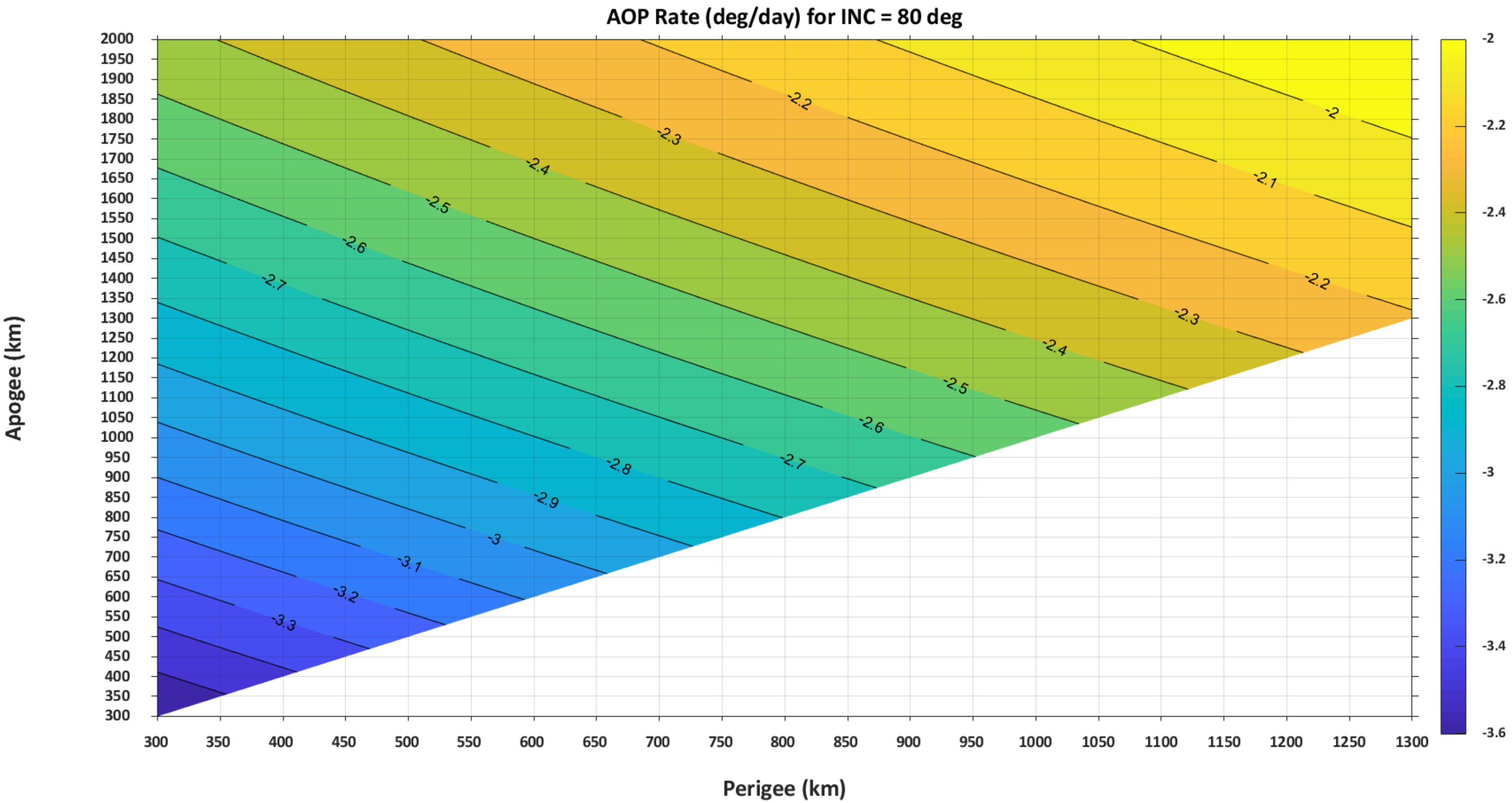
where

- R_E – Earth Radius
- n – mean motion ($\sqrt{\mu/a^3}$)
- J_2 – Geopotential term related to Earth's oblateness

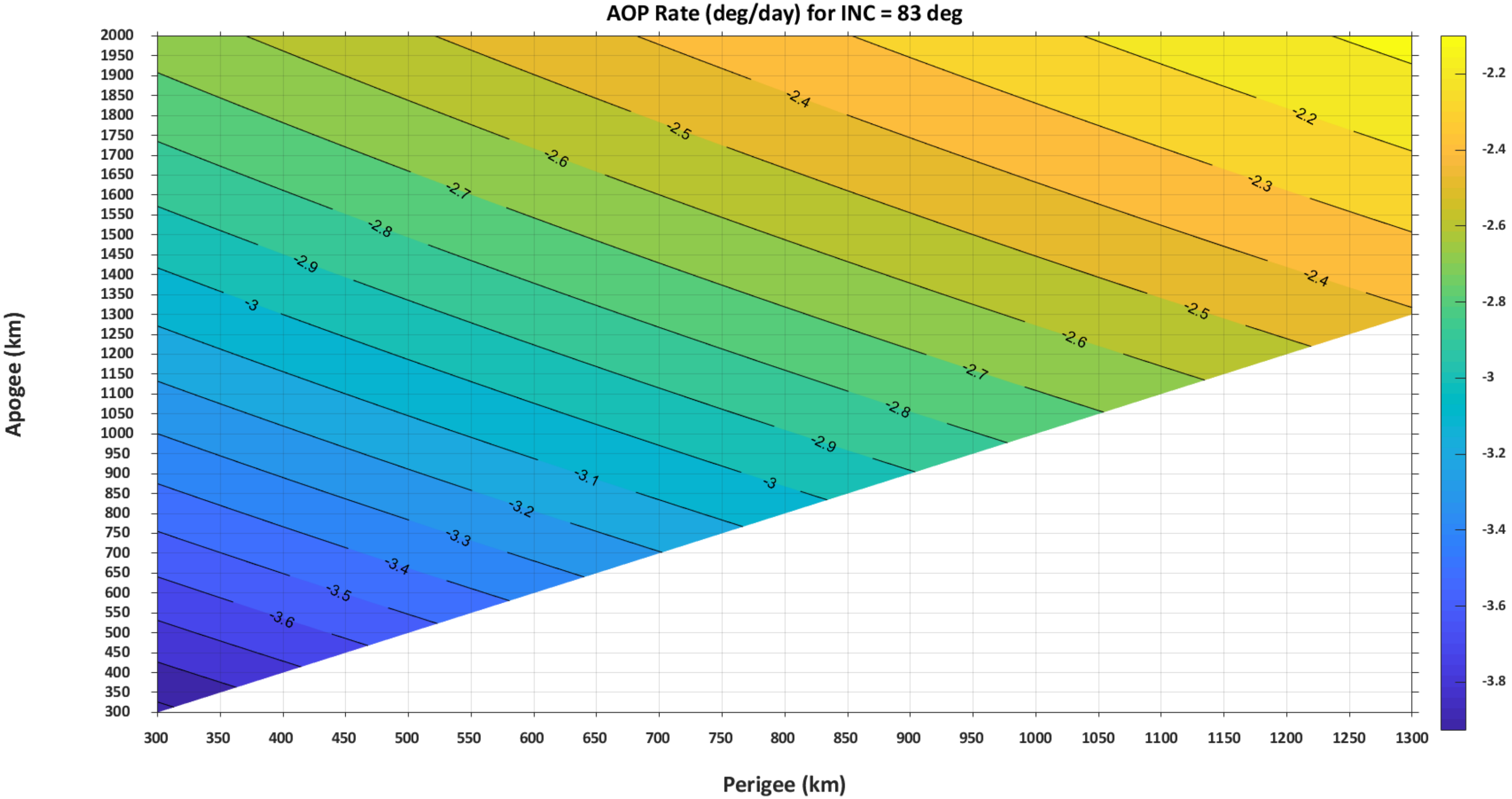


- Perigee and apogee will precess in latitude over time

Apsidal Precession - 80° Inclination



Apsidal Precession - 83° Inclination



Differential Precession

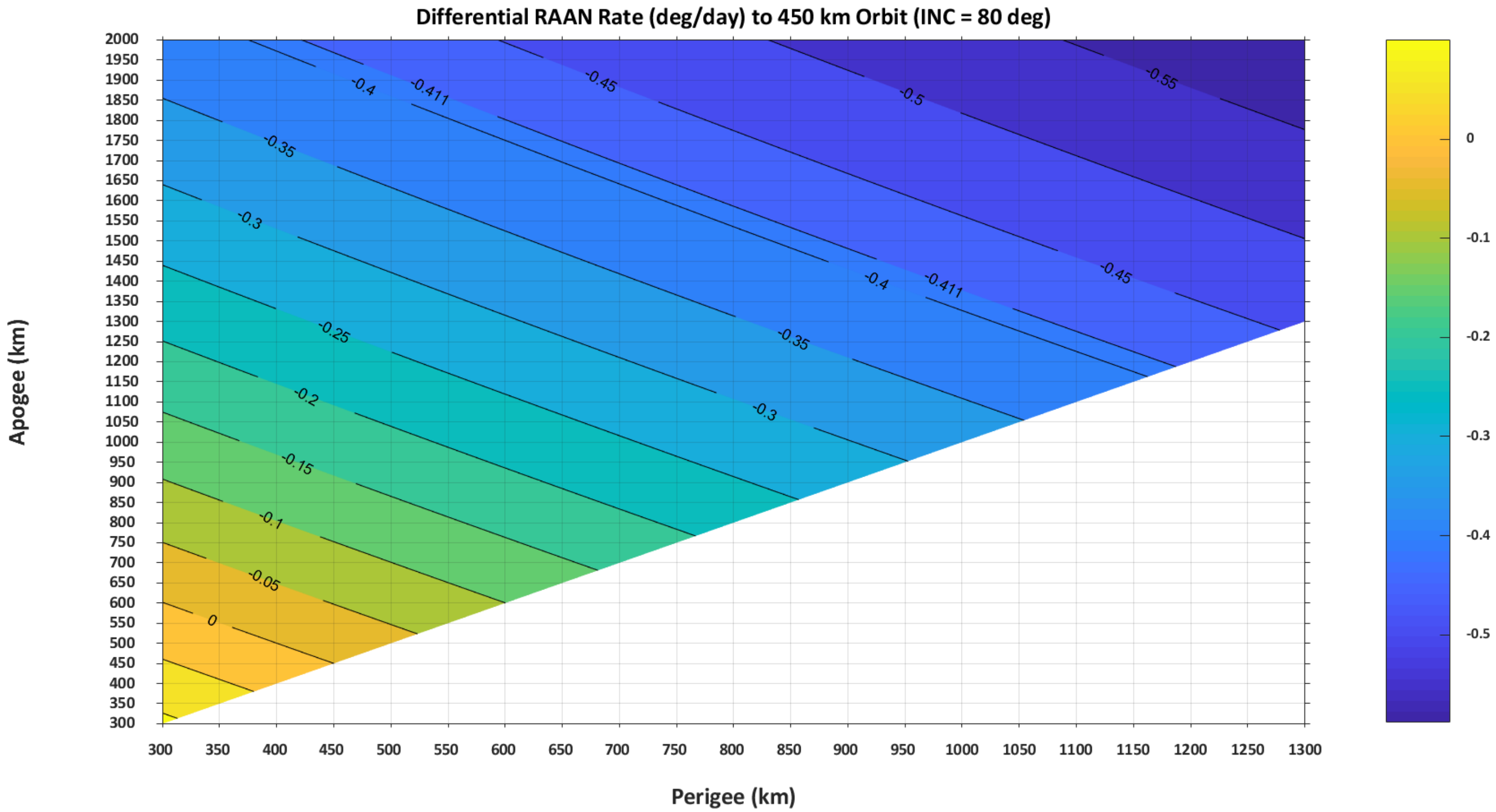
- A mission designer can use differential precession to space orbit planes apart from each other

- This can be achieved by maneuvering 1 satellite into a different orbit by changing apogee or perigee and allowing it to drift relative to the other satellite(s)

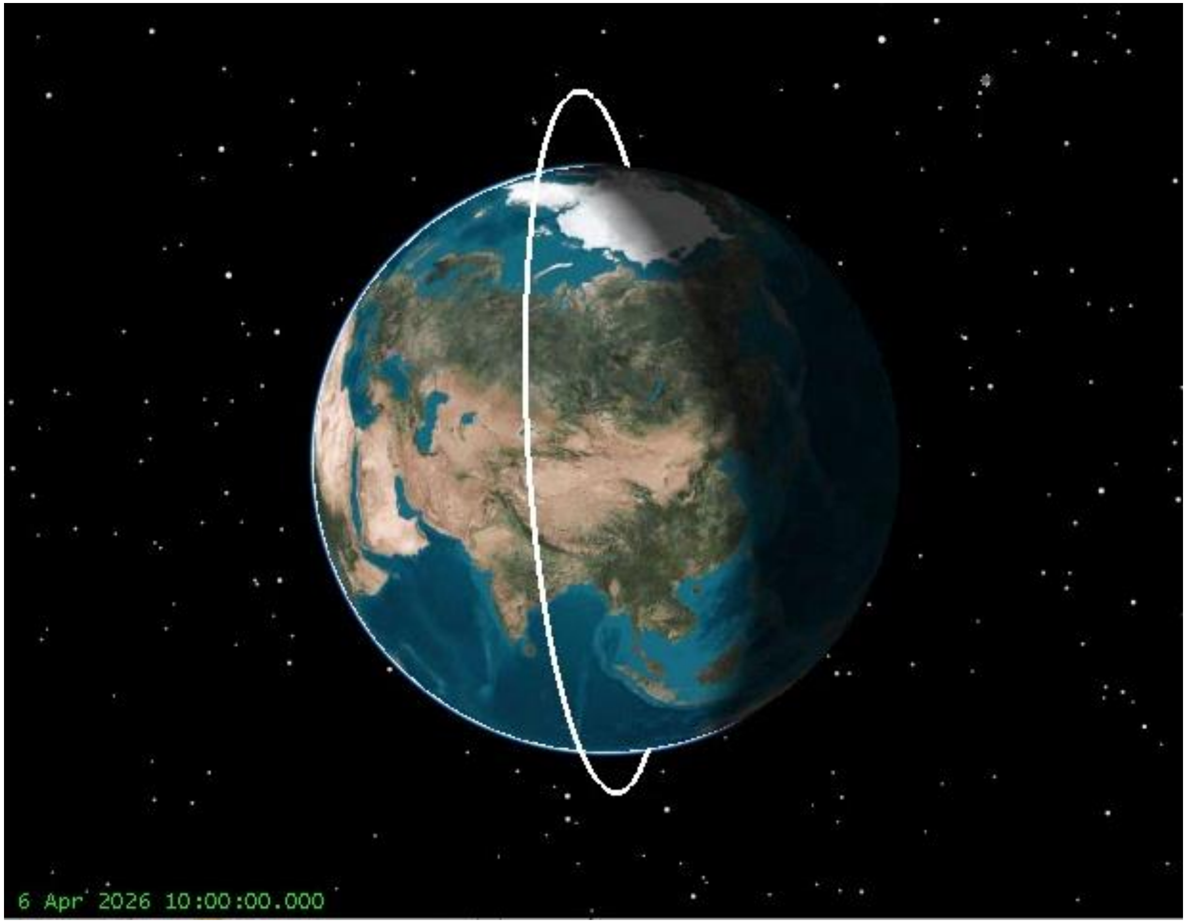
- However, the precession is in the Ascending Node – the inclination remains the same
  - Fuel costs for inclination changes are incredibly high

Differential Precession Example

- In this example, we can see the differential nodal drift relative to a 450 km circular orbit with an inclination of 80°



Differential Precession Movie

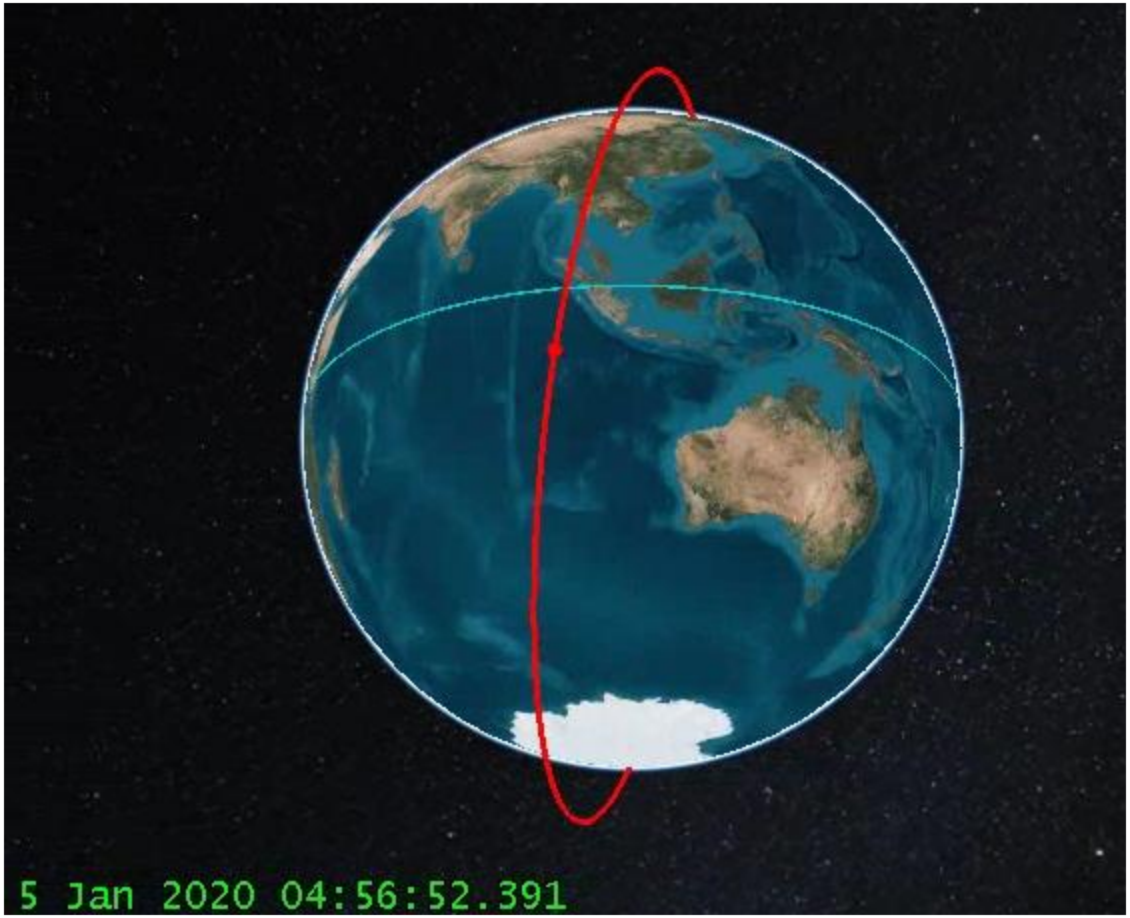


Nodal/Apsidal Interactions

- The differences in the nodal and apsidal precession rates can create interesting observation opportunities
  - The perigee latitude changes as the argument of perigee (ω) precesses
  - The perigee local time changes as the ascending node (Ω) precesses. Furthermore, the local time will ‘flip’ by 12 hours as perigee passes over the pole

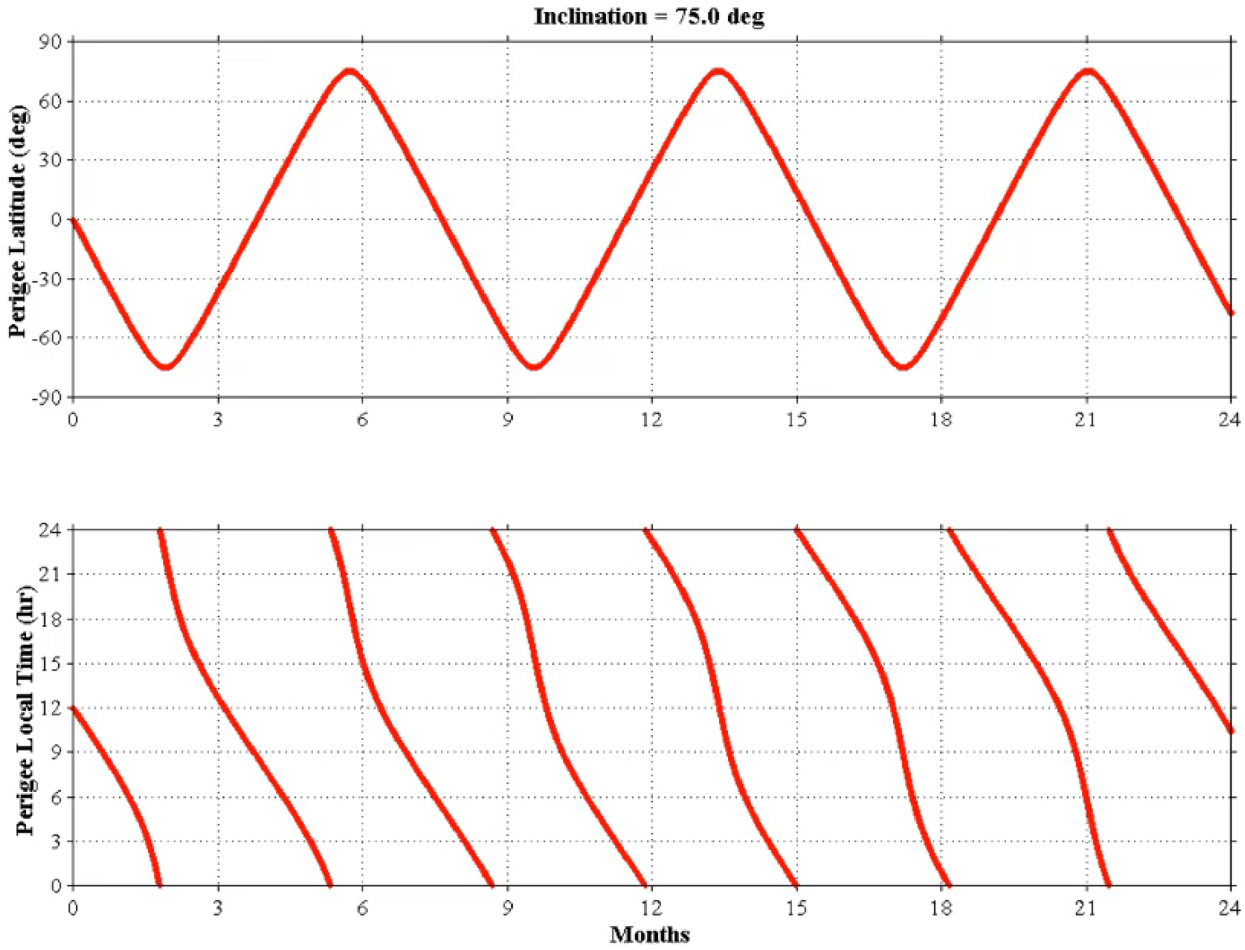
- An example of this feature is shown on the following slides for a 200 x 2000 km orbit with varying inclination, simulated 2 years
  - The plots show the Latitude and Local Solar Time of the perigee position

Precession Movie



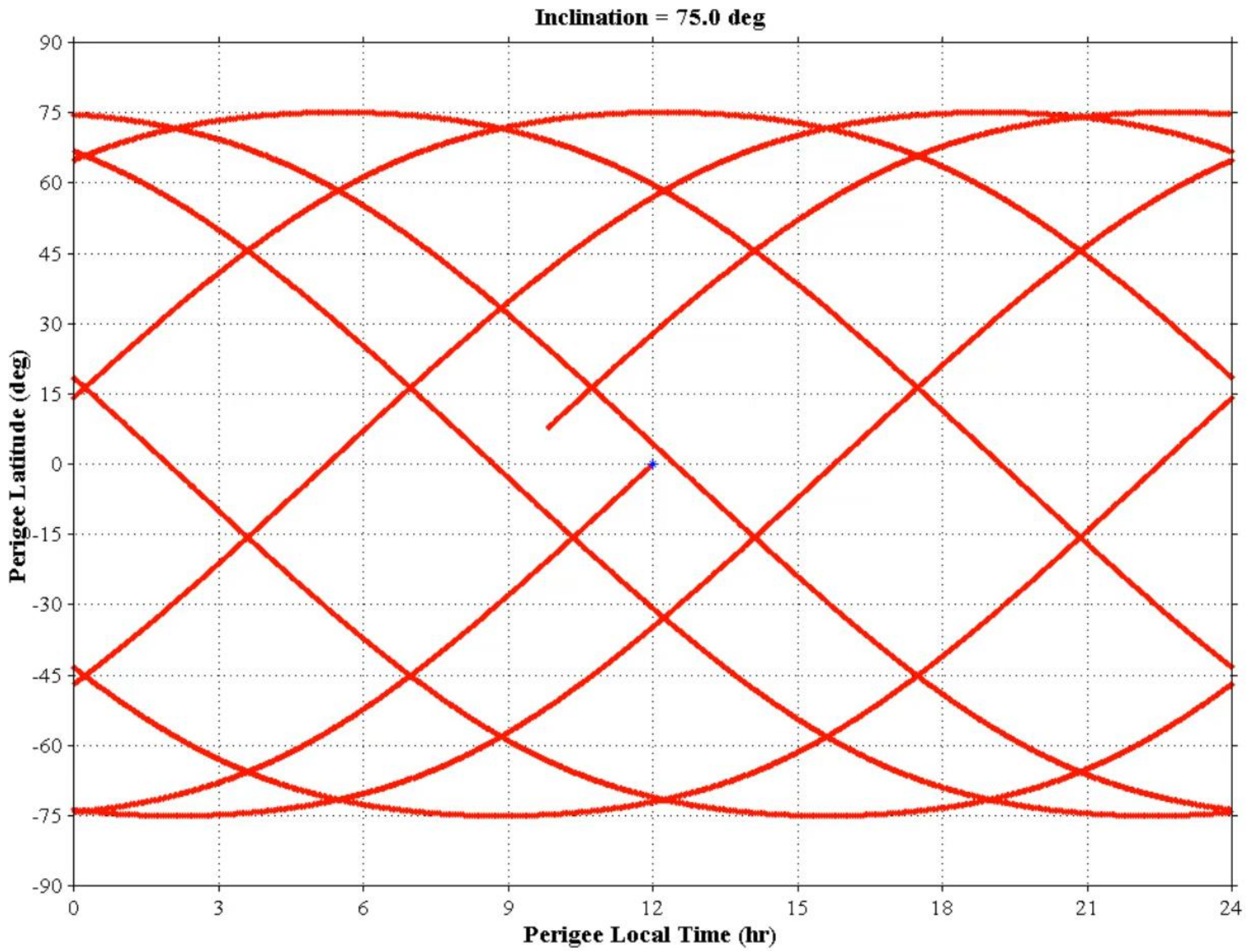
Perigee Latitude & Local Time vs. Time

- Over this inclination range (75 – 80 deg)
  - Absolute apsidal rates increase as inclination increases
  - Absolute nodal rate decrease as inclination increases



Perigee Latitude vs Local Time

- Judicious selection of orbit parameters and inclination can yield interesting patterns
  - This example has a “Frozen Orbit” at an inclination near 77.5 80, and 83.5 deg

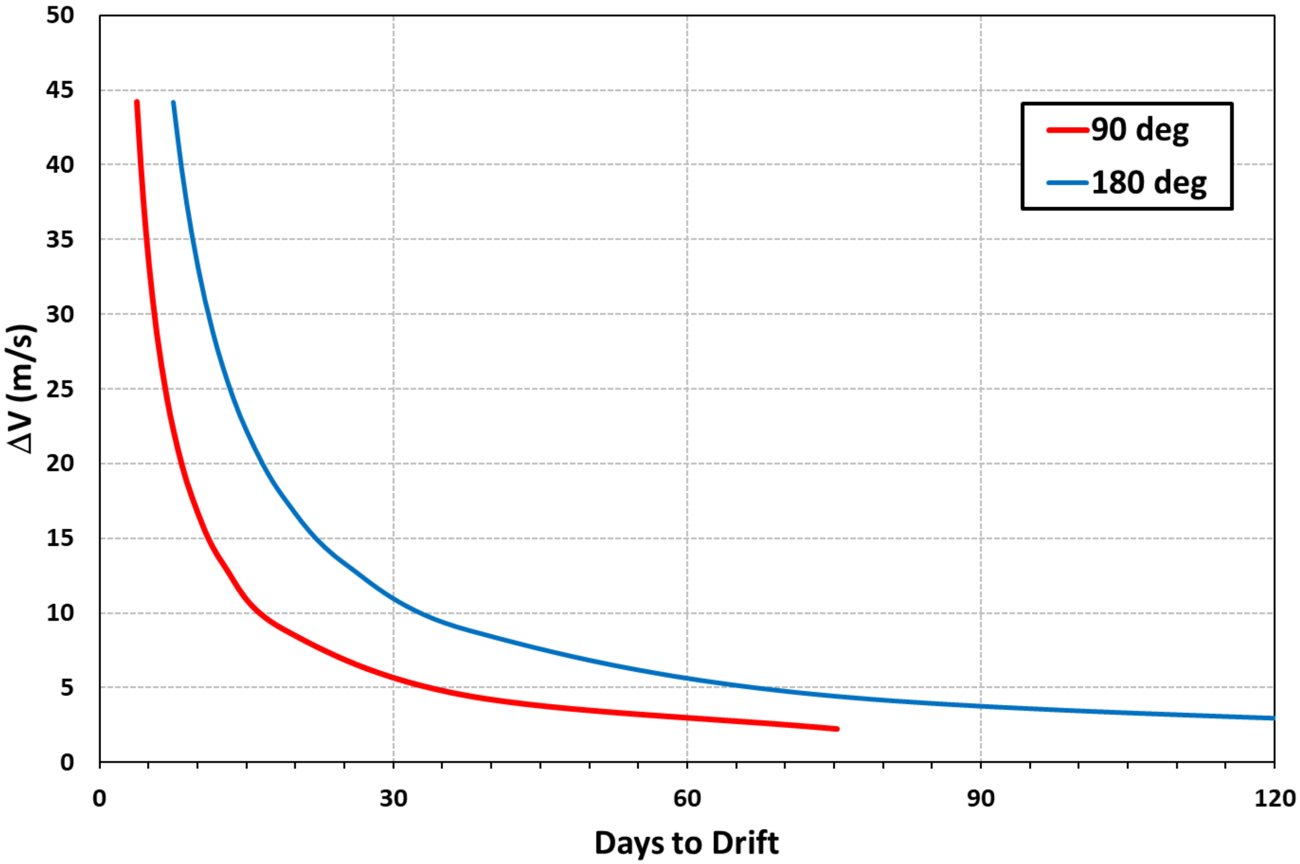


String of Pearls

- Launching multiple spacecraft and spacing them out in a “string of pearls” is a simple operation
- A small maneuver(s) can be performed to change the orbital period slightly to allow for “in-plane” drift
- Once the desired separation has been achieved then maneuver(s) can be performed to stop the drift
- Ultimately, the cost of the drift maneuvers is dependent on how long is allowable to achieve the desired separation

In-Plane Drift

- As requested, the ΔV cost vs. time to drift was computed for a 500 km orbit
- The quicker the drift, the more ΔV is necessary



Plane Change ΔV

- Plane changes are proportional to the orbital velocity

  $$\Delta V = 2V_i \sin(\frac{\Delta\alpha}{2})$$

- For a satellite flying in a 500 km orbit (Vcirc = 7.613 km/s) the cost to change inclination by 1 deg is **133 m/s**

- Nodal rotations are slightly more complicated as the orbital plane change is a function of both the inclination and the nodal change (using spherical trigonometry)

  $$\cos\alpha = \cos i_i \cos i_f + \sin i_i \sin i_f \cos(\Delta\Omega)$$

  i_i & i_f – initial/final inclination
  ΔΩ – nodal change

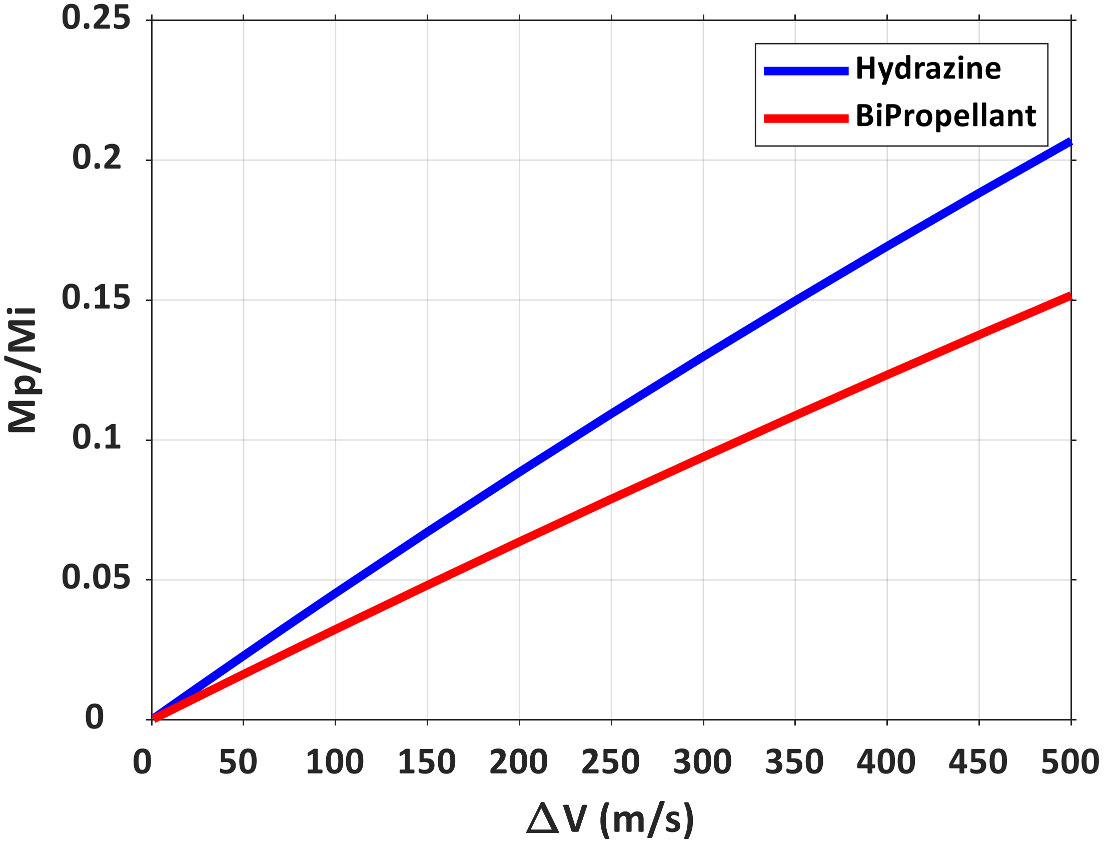
- Using the previous example, a 30 deg nodal rotation in a 80 deg inclination orbit (without changing inclination) costs **3881 m/s!!!**
- Plane changes in low-Earth orbit are expensive

ΔV & Propellant Calculations

- Propellant mass fraction is a function of ΔV and propellant specific impulse (Isp)

$$\frac{M_P}{M_i} = 1 - e^{-\Delta V/gI_{sp}}$$

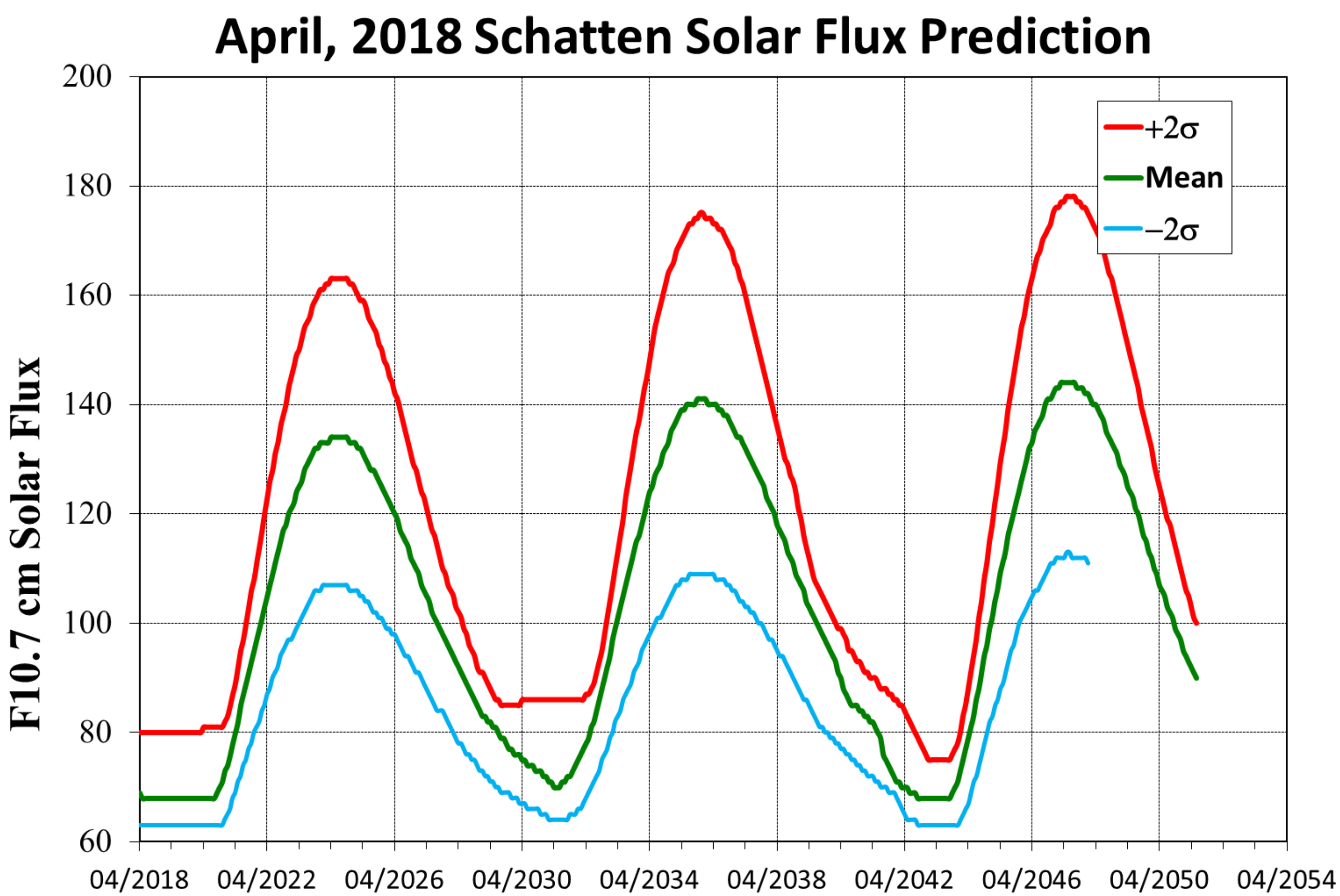
- Common Isp values
  - Hydrazine: 220 sec
  - BiPropellant: 310 sec



Orbital Decay

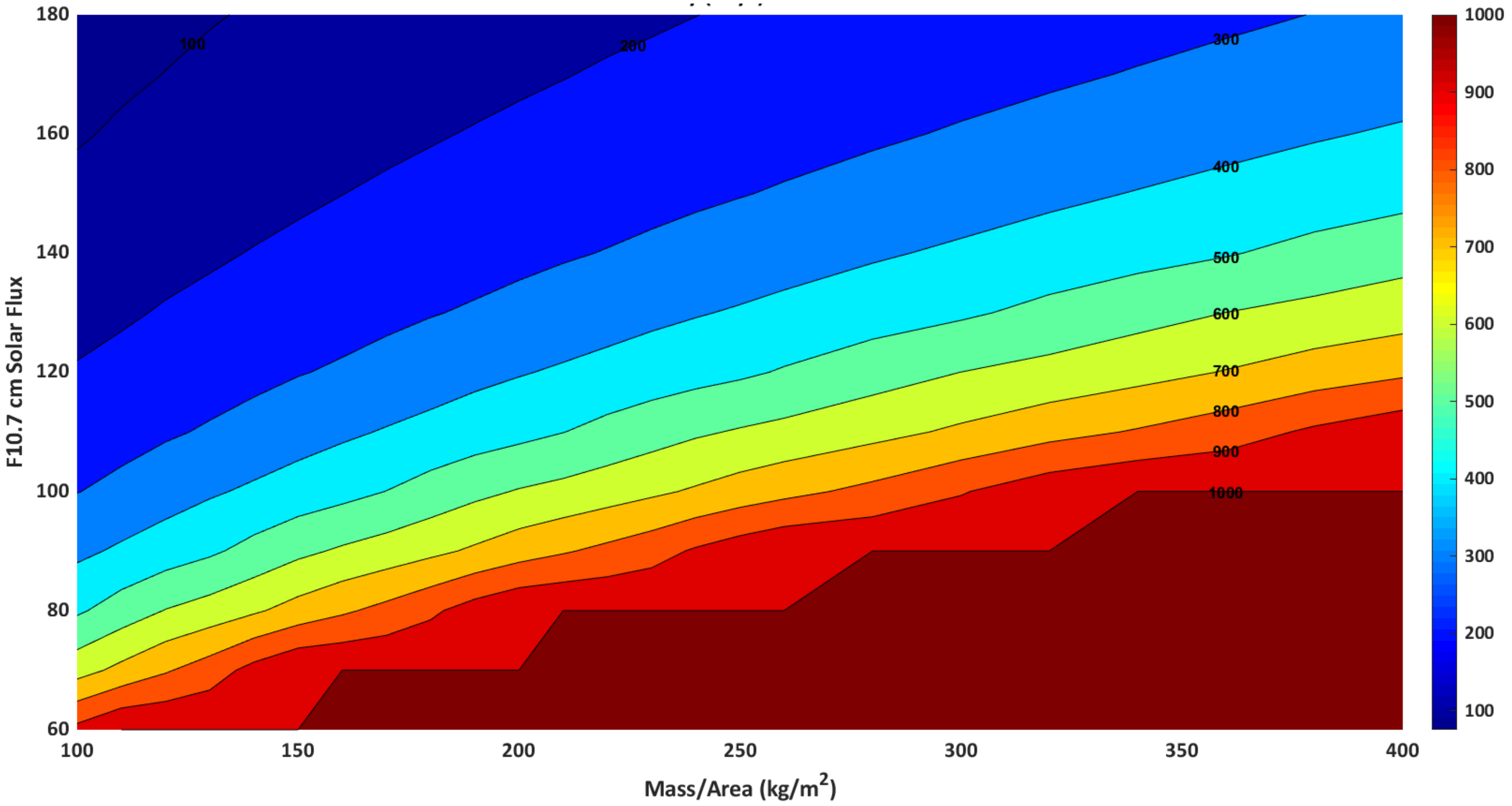
- Orbital decay is a function of Solar Activity and spacecraft ballistic properties
- Solar activity is obtained via solar flux predictions
- The spacecraft ballistic properties are a function of the Mass/Area ratio
  - The larger Mass/Area ratio, the more resistant the spacecraft is to atmospheric drag, the slower the orbital decay
  - Need to take into account all appendages (e.g. solar arrays, booms, appendages) as well as the attitude profile when computing the drag area
- As mass is expended (from thruster firings) the spacecraft becomes less resistant to atmospheric drag
- Because of these variables, it's difficult to generalize on lifetime durations

Solar Flux Prediction



Orbit Decay (days) from 450 to 425 km

- Decay times range from 3 months to 3 years



Radiation Dosage

- David Batchelor (Code 586) computed the Total Ionizing Dose (TID) for the orbit regimes that were requested and the results are below

Orbit	300 km	700 km	300 x 1200 km
TID (krad)	2.4	6.0	11.9

- Assumptions
  - Shield Thickness: 100 mil Al
  - Orbit Inclination: 82 deg
  - Lifetime: 1 year
  - Launch Date: 1/1/2023 (Solar Max)
    - ✓ Solar minimum conditions reduces TID by 38%, 17%, and 8%, respectively