Upper Division

Lecture Tutorial: Modeling the Sun-Earth-Moon System

Description: This guided inquiry paper-and-pencil activity helps students to understand in detail the motion of the three-body system that consists of the Sun, Earth, and Moon. This resource is designed to supplement *Lecture-Tutorials for Introductory Astronomy* for lecture-style classrooms.

Prerequisite:

- Understand the orbital frequency of the Earth and Moon.

This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

Learning Sequence:

I. Plotting the positions of the Earth and Moon over the course of one year

In this tutorial we explore in detail the motion of the three-body system that consists of the Sun, Earth, and Moon. We begin by constructing a diagram mapping the motion of the Earth and Moon during the course of a year. To start with, we will make theses simplifying approximations:

- The Sun-Earth distance remains essentially constant, as does the Earth-Moon distance. *(That is, we ignore the eccentricities of Earth's orbit around the Sun and the Moon's orbit around Earth.)*
- The Moon completes 12 revolutions around Earth during a single year. *(The synodic month is actually fewer than 30 days long.)*
- The Sun, Earth, and Moon are always located in the imaginary plane containing the Earth's orbit around the Sun. *(In actuality, the Moon's orbit around Earth is "tilted" by about 5° relative to the ecliptic plane.)*

A. Figure 1 on the enlargement provided shows a large circle divided into 12 sections. The center of the circle will represent the location of the Sun. The circle itself will, for now, represent the Earth's orbit.

With your partner(s), plot the locations of the Earth and Moon at **half-month intervals**, with each plot showing the relative positions of the Moon and Earth during either (i) a full moon phase or (ii) a new moon phase.

B. Using the points you have plotted, sketch as best you can what must be the path taken by the Moon over the course of a year. How would you describe the shape of this path in your own words?

✓ **Please STOP HERE** to check your work thus far with an instructor before continuing on.

This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

II. Refining our model of the Sun-Earth-Moon system

Your sketch of the Moon's path around the Sun may surprise you! In the remainder of this tutorial we will examine the Moon's behavior in greater detail. To do so, we will need the following data:

Mean distance Sun-Earth distance:	R_E	$= 1.50 \times 10^{11}$ m
Mean distance Earth-Moon distance:	R_M	$= 3.84 \times 10^{8}$ m
Mass of Sun:	M_{Sun}	$= 1.99 \times 10^{30}$ kg
Mass of Earth:	M_{Earth}	$= 5.98 \times 10^{24}$ kg
Mass of Earth:	M_{Moon}	$= 7.36 \times 10^{22}$ kg
Universal gravitational constant:	G	$= 6.67 \times 10^{-11}$ N-m^2/kg^2

A. Figure 1 from the enlargement included a circle of about 8.0 cm in radius to represent the Earth's orbit. How many centimeters must we use for the (mean) Earth-Moon distance if we were to redraw that diagram *to scale?* Show all work. (Would drawing the diagram to scale even be possible?)

B. Let's now apply Newton's Second Law and Newton's Law of Gravitation to consider some details regarding the shape of path you sketched in Section I above.

1. Using the astronomical data shown above, compute the net force exerted on the Moon—magnitude and direction—for the case in which:

- the Moon is in a *full moon* phase

- the Moon is in a *new moon* phase

This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

2. Now go back to the enlargement (used in Section I) on which you and your partners plotted the positions of the Earth and Moon for several instances of full moon and new moon.

   Consider a point along the Moon's path when it undergoes a **full moon** phase. On the basis of your results in part 1 above, what can be said about the curvature of that part of the path? (That is, does that part of the Moon's path curve "toward" or "away from" the Sun?) Explain.

   Now answer this same question for a **new moon** phase: Does that part of the Moon's path curve "toward" or "away from" the Sun? Explain your reasoning.

3. Turn the enlargement over to have Figure 2 showing. This figure shows an arc length of approximately 30° representing part of the Earth's orbit around the Sun. The two small dots along the arc (about 15° apart) represent the location of Earth along its orbit at consecutive new moon and full moon phases.

   With your partners, summarize your results from this part of the tutorial by carefully sketching a more refined diagram showing the path of the Moon. Be sure that your sketch takes into account as best you can:

   - the relative sizes of the (mean) Sun-Earth and Earth-Moon distances

   - the curvature of the Moon's path

✓ **Please STOP HERE** to check your work thus far with an instructor before continuing on.

This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

C. Finally, we can make one further refinement to our model when considering the Earth and Moon together as a unit.

1. Use the astronomical data from p. 1 to determine the location of the **center of mass** of the system consisting of the Earth and Moon. Discuss your reasoning with your partners and show all work.

Is the center of mass of this system located at the exact center of the Earth? If not, is it located somehwere within the interior of the Earth? (Use 6.37×10^6 m for the mean radius of the Earth.)

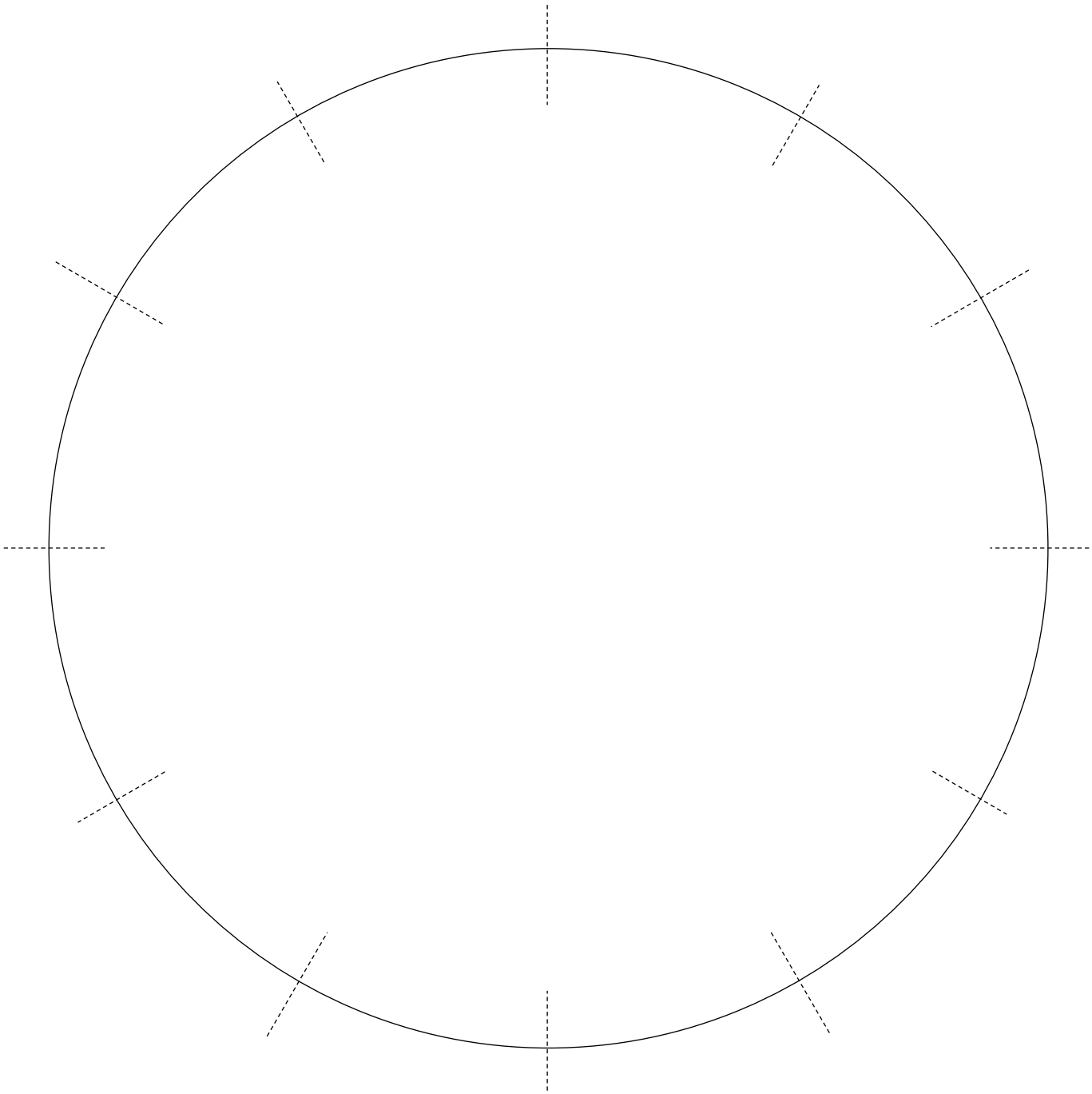
2. Reflect upon the work you and your partners did on both Fig. 1 and Fig. 2 of the enlargement. Given your results here in part C, is it accurate to say that the circle on Fig. 1 (or the circular arc on Fig. 2) should represent the path taken by Earth as it travels around the Sun?

- If so, explain why.

- If not, explain why not, and decide with your partners what the circle (and circular arc) in Fig. 1 (and Fig. 2) should represent.

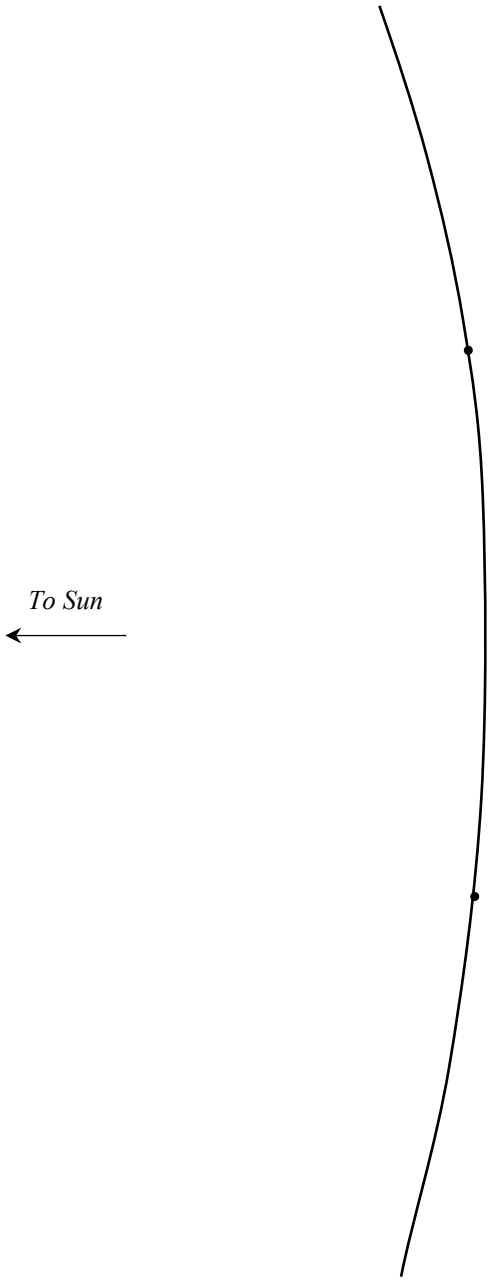
This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

Figure 1: Plotting the positions of the Earth and Moon over the course of one year (Section I)



This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.

Figure 2: Refining our model of the Sun-Earth-Moon system (Section II.B)



This resource was developed by B. Ambrose, X. Cid, and R. Lopez. The co-authors acknowledge useful discussions with J. Bailey, R. Vieyra, and S. Willoughby, and the support of a subcontract from the NASA Heliophysics Education Activation Team to Temple University and the AAPT under NASA Grant/Cooperative Agreement Number NNX16AR36A.